

The corrections from one loop and two-loop Barr-Zee type diagrams to muon MDM in BLMSSM

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Abstract

In a supersymmetric extension of the standard model where baryon and lepton numbers are local gauge symmetries (BLMSSM) and the Yukawa couplings between Higgs doublets and exotic quarks are considered, we study the one loop diagrams and the two-loop Barr-Zee type diagrams with a closed Fermi(scalar) loop between the vector Boson and Higgs. Using the effective Lagrangian method, we deduce the Wilson coefficients of dimension 6 operators contributing to the anomalous magnetic moment of muon, which satisfies the electromagnetic gauge invariance. In the numerical analysis, we consider the experiment constraints from Higgs and neutrino data. In some parameter space, the new physics contribution is large and even reaches 24×10^{-10} , which can remedy the deviation well.

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I. INTRODUCTION

The magnetic dipole moment (MDM) of lepton has close relation with the new physics beyond the standard model(SM). The current world average value [1] of $(g-2)_\mu$ experiment is

$$a_\mu^{exp} = \frac{1}{2}(g_\mu - 2) = 11659208.9(5.4)(3.3) \times 10^{-10}. \quad (1)$$

There are three type contributions to the MDM of muon [2] such as: QED loops, hadronic contributions and electroweak corrections. The SM theoretical prediction of muon MDM is[3]

$$a_\mu^{SM} = 11659184.1(4.8) \times 10^{-10}. \quad (2)$$

The deviation between the SM prediction and experimental result is given as the follows, which lies in the range of $\sim 3\sigma$ [4].

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 24.8(8.7)(4.8) \times 10^{-10}. \quad (3)$$

The minimal supersymmetric extension of the standard model (MSSM) [5] is one of the most attractive candidates in the models beyond the SM, and draws physicists most attentions for a long time. A minimal supersymmetric extension of the SM with local gauged B and L(BLMSSM) is a favorite one, because it has two advantages. 1. The broken baryon number (B) can explain asymmetry of matter-antimatter in the universe. 2. The neutrinos should have tiny mass from the neutrino oscillation experiment. In theory, the tiny mass can be induced from the heavy majorana neutrinos by the seesaw mechanism. Therefore, at some scale the lepton number (L) should be broken too.

Extending SM, with B and L as spontaneously broken gauge symmetries around TeV scale the models are studied[6]. Neglecting the Yukawa couplings between Higgs doublets and exotic quarks in BLMSSM, the authors study the lightest CP-even Higgs [6, 7]. In the BLMSSM, considering the Yukawa couplings between Higgs and exotic quarks, we study the lightest CP-even Higgs(h^0) mass and the decays $h^0 \rightarrow \gamma\gamma$, $h^0 \rightarrow ZZ(WW)$ [8], which are also studied in other models. In the CP-violating BLMSSM, the neutron electric dipole moment(EDM) is investigated[9].

To find new physics beyond the SM, research the MDMs [10, 11] and EDMs[12] of leptons are the effective ways. There are some works for the supersymmetric (SUSY) one-loop

contributions to muon MDM, and in some parameter space[13] the numerical results can be large. In $\mu\nu MSSM$, we study the muon MDM at one-loop level[4]. The authors investigate two-loop Barr-Zee-type diagrams[14] and obtain the electric dipole moments (EDMs) and MDMs of light fermions. Using the heavy mass expansion approximation (HME) and the projection operator method, the authors show two-loop standard electroweak corrections to muon MDM [15]. There are also several works about the muon MDM from two-loop diagrams [16, 17] in SUSY model.

In this work, we study the one loop diagrams and two-loop Barr-Zee type diagrams with a closed scalar (Fermi) loop between vector Boson and Higgs in the frame work of BLMSSM. Taking into account the Yukawa couplings between Higgs doublets and exotic quarks, we investigate these contributions to muon MDM with the effective Lagrangian method. Using the same method as in the Ref.[17], we deduce all dimension 6 operators and their coefficients. Attaching a photon in all possible ways on the internal line of one self-energy diagram, one can obtain the corresponding triangle diagrams, and the sum of these amplitudes satisfies the Ward identity required by the QED gauge symmetry. Adopting the equations of motion to external leptons, we can neglect higher dimensional operators(dimension 8 operators) safely.

After this introduction, we briefly summarize the main ingredients of the BLMSSM, and show the needed couplings for exotic leptons and exotic quarks in section 2. We collect the one-loop and two-loop corrections to the muon MDM in section 3. Section 4 is devoted to the numerical analysis and discussion for the dependence of muon MDM on the BLMSSM parameters. In section 5, we give our conclusion. Some formulae are collected in the appendix.

II. SOME COUPLING IN BLMSSM

Physicists enlarge the SM with the local gauge group of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$, and obtain BLMSSM [6]. To to cancel L and B anomaly, the exotic leptons ($\hat{L}_4 \sim (1, 2, -1/2, 0, L_4)$, $\hat{E}_4^c \sim (1, 1, 1, 0, -L_4)$, $\hat{N}_4^c \sim (1, 1, 0, 0, -L_4)$, $\hat{L}_5^c \sim (1, 2, 1/2, 0, -(3 + L_4))$, $\hat{E}_5 \sim (1, 1, -1, 0, 3 + L_4)$, $\hat{N}_5 \sim (1, 1, 0, 0, 3 + L_4)$) and the exotic quarks ($\hat{Q}_4 \sim (3, 2, 1/6, B_4, 0)$, $\hat{U}_4^c \sim (\bar{3}, 1, -2/3, -B_4, 0)$, $\hat{D}_4^c \sim (\bar{3}, 1, 1/3, -B_4, 0)$, $\hat{Q}_5^c \sim (\bar{3}, 2, -1/6, -(1 + B_4), 0)$, $\hat{U}_5 \sim (3, 1, 2/3, 1 + B_4, 0)$,

$\hat{D}_5 \sim (3, 1, -1/3, 1 + B_4, 0)$ are respectively introduced. The detection of the lightest CP even Higgs h^0 at LHC[18] makes people to be convinced of the Higgs mechanism. To break lepton number and baryon number spontaneously, the Higgs superfields $\hat{\Phi}_L, \hat{\varphi}_L$ and $\hat{\Phi}_B, \hat{\varphi}_B$ are introduced respectively, and they acquire nonzero vacuum expectation values (VEVs). The exotic quarks are very heavy and unstable. So the superfields \hat{X}, \hat{X}' are also introduced in the BLMSSM and the lightest superfields X can be a candidate for dark matter.

The superpotential of BLMSSM is[8]

$$\begin{aligned}
\mathcal{W}_{BLMSSM} &= \mathcal{W}_{MSSM} + \mathcal{W}_B + \mathcal{W}_L + \mathcal{W}_X , \\
\mathcal{W}_B &= \lambda_Q \hat{Q}_4 \hat{Q}_5^c \hat{\Phi}_B + \lambda_U \hat{U}_4^c \hat{U}_5 \hat{\varphi}_B + \lambda_D \hat{D}_4^c \hat{D}_5 \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B \\
&\quad + Y_{u_4} \hat{Q}_4 \hat{H}_u \hat{U}_4^c + Y_{d_4} \hat{Q}_4 \hat{H}_d \hat{D}_4^c + Y_{u_5} \hat{Q}_5^c \hat{H}_d \hat{U}_5 + Y_{d_5} \hat{Q}_5^c \hat{H}_u \hat{D}_5 , \\
\mathcal{W}_L &= Y_{e_4} \hat{L}_4 \hat{H}_d \hat{E}_4^c + Y_{\nu_4} \hat{L}_4 \hat{H}_u \hat{N}_4^c + Y_{e_5} \hat{L}_5^c \hat{H}_u \hat{E}_5 + Y_{\nu_5} \hat{L}_5^c \hat{H}_d \hat{N}_5 \\
&\quad + Y_\nu \hat{L} \hat{H}_u \hat{N}^c + \lambda_{N^c} \hat{N}^c \hat{N}^c \hat{\varphi}_L + \mu_L \hat{\Phi}_L \hat{\varphi}_L , \\
\mathcal{W}_X &= \lambda_1 \hat{Q} \hat{Q}_5^c \hat{X} + \lambda_2 \hat{U}^c \hat{U}_5 \hat{X}' + \lambda_3 \hat{D}^c \hat{D}_5 \hat{X}' + \mu_X \hat{X} \hat{X}' .
\end{aligned} \tag{4}$$

where \mathcal{W}_{MSSM} is the superpotential of the MSSM. The soft breaking terms \mathcal{L}_{soft} of the BLMSSM can be written in the following form[8].

$$\begin{aligned}
\mathcal{L}_{soft} &= \mathcal{L}_{soft}^{MSSM} - (m_{\tilde{\nu}^c}^2)_{IJ} \tilde{N}_I^{c*} \tilde{N}_J^c - m_{\tilde{Q}_4}^2 \tilde{Q}_4^\dagger \tilde{Q}_4 - m_{\tilde{U}_4}^2 \tilde{U}_4^{c*} \tilde{U}_4^c - m_{\tilde{D}_4}^2 \tilde{D}_4^{c*} \tilde{D}_4^c \\
&\quad - m_{\tilde{Q}_5}^2 \tilde{Q}_5^{c\dagger} \tilde{Q}_5^c - m_{\tilde{U}_5}^2 \tilde{U}_5^* \tilde{U}_5 - m_{\tilde{D}_5}^2 \tilde{D}_5^* \tilde{D}_5 - m_{\tilde{L}_4}^2 \tilde{L}_4^\dagger \tilde{L}_4 - m_{\tilde{\nu}_4}^2 \tilde{N}_4^{c*} \tilde{N}_4^c \\
&\quad - m_{\tilde{e}_4}^2 \tilde{E}_4^{c*} \tilde{E}_4^c - m_{\tilde{L}_5}^2 \tilde{L}_5^{c\dagger} \tilde{L}_5^c - m_{\tilde{\nu}_5}^2 \tilde{N}_5^* \tilde{N}_5 - m_{\tilde{e}_5}^2 \tilde{E}_5^* \tilde{E}_5 - m_{\Phi_B}^2 \Phi_B^* \Phi_B \\
&\quad - m_{\varphi_B}^2 \varphi_B^* \varphi_B - m_{\Phi_L}^2 \Phi_L^* \Phi_L - m_{\varphi_L}^2 \varphi_L^* \varphi_L - (m_B \lambda_B \lambda_B + m_L \lambda_L \lambda_L + h.c.) \\
&\quad + \{ A_{u_4} Y_{u_4} \tilde{Q}_4 H_u \tilde{U}_4^c + A_{d_4} Y_{d_4} \tilde{Q}_4 H_d \tilde{D}_4^c + A_{u_5} Y_{u_5} \tilde{Q}_5^c H_d \tilde{U}_5 + A_{d_5} Y_{d_5} \tilde{Q}_5^c H_u \tilde{D}_5 \\
&\quad + A_{BQ} \lambda_Q \tilde{Q}_4 \tilde{Q}_5^c \Phi_B + A_{BU} \lambda_U \tilde{U}_4^c \tilde{U}_5 \varphi_B + A_{BD} \lambda_D \tilde{D}_4^c \tilde{D}_5 \varphi_B + B_B \mu_B \Phi_B \varphi_B + h.c. \} \\
&\quad + \{ A_{e_4} Y_{e_4} \tilde{L}_4 H_d \tilde{E}_4^c + A_{\nu_4} Y_{\nu_4} \tilde{L}_4 H_u \tilde{N}_4^c + A_{e_5} Y_{e_5} \tilde{L}_5^c H_u \tilde{E}_5 + A_{\nu_5} Y_{\nu_5} \tilde{L}_5^c H_d \tilde{N}_5 \\
&\quad + A_N Y_\nu \tilde{L} H_u \tilde{N}^c + A_{N^c} \lambda_{N^c} \tilde{N}^c \tilde{N}^c \varphi_L + B_L \mu_L \Phi_L \varphi_L + h.c. \} \\
&\quad + \{ A_1 \lambda_1 \tilde{Q} \tilde{Q}_5^c X + A_2 \lambda_2 \tilde{U}^c \tilde{U}_5 X' + A_3 \lambda_3 \tilde{D}^c \tilde{D}_5 X' + B_X \mu_X X X' + h.c. \} ,
\end{aligned} \tag{5}$$

The $SU(2)_L$ singlets $\Phi_B, \varphi_B, \Phi_L, \varphi_L$ and the $SU(2)_L$ doublets H_u, H_d should obtain nonzero VEVs $v_B, \bar{v}_B, v_L, \bar{v}_L$ and v_u, v_d respectively. Therefore, the local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ breaks down to the electromagnetic symmetry $U(1)_e$.

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + H_u^0 + iP_u^0) \end{pmatrix} , \quad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + H_d^0 + iP_d^0) \\ H_d^- \end{pmatrix} ,$$

$$\begin{aligned}
\Phi_B &= \frac{1}{\sqrt{2}}(v_B + \Phi_B^0 + iP_B^0), & \varphi_B &= \frac{1}{\sqrt{2}}(\bar{v}_B + \varphi_B^0 + i\bar{P}_B^0), \\
\Phi_L &= \frac{1}{\sqrt{2}}(v_L + \Phi_L^0 + iP_L^0), & \varphi_L &= \frac{1}{\sqrt{2}}(\bar{v}_L + \varphi_L^0 + i\bar{P}_L^0),
\end{aligned} \tag{6}$$

In Ref.[8], the mass matrixes of Higgs, exotic quarks and exotic scalar quarks are obtained. Some mass matrixes of exotic scalar leptons are discussed by the authors [19]. Here, we show the mass matrixes of exotic scalar leptons in our notation. Because the super fields \hat{N}^c are introduced in BLMSSM, the neutrinos can have tiny masses, and the scalar neutrinos are double as those in MSSM.

A. The mass matrix

After symmetry breaking the mass matrix for neutrinos in the left-handed basis (ν, N^c) is given by the following matrix.

$$-\mathcal{L}_{mass}^\nu = (\bar{\nu}_R^I, \bar{N}_R^{cI}) \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}}(Y_\nu)^{IJ} \\ \frac{v_u}{\sqrt{2}}(Y_\nu^T)^{IJ} & \frac{\bar{v}_L}{\sqrt{2}}(\lambda_{N^c})^{IJ} \end{pmatrix} \begin{pmatrix} \nu_L^J \\ N_L^{cJ} \end{pmatrix} + h.c. \tag{7}$$

Using the unitary transformations

$$\begin{pmatrix} \nu_{1L}^I \\ \nu_{2L}^I \end{pmatrix} = U_{\nu^{IJ}}^\dagger \begin{pmatrix} \nu_L^J \\ N_L^{cJ} \end{pmatrix}, \quad \begin{pmatrix} \nu_{1R}^I \\ \nu_{2R}^I \end{pmatrix} = W_{\nu^{IJ}}^\dagger \begin{pmatrix} \nu_R^J \\ N_R^{cJ} \end{pmatrix}, \tag{8}$$

we diagonalize the mass matrix for neutrinos:

$$W_{\nu^{IJ}}^\dagger \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}}(Y_\nu)^{IJ} \\ \frac{v_u}{\sqrt{2}}(Y_\nu^T)^{IJ} & \frac{\bar{v}_L}{\sqrt{2}}(\lambda_{N^c})^{IJ} \end{pmatrix} U_{\nu^{IJ}} = \text{diag}(m_{\nu_1^I}, m_{\nu_2^I}). \tag{9}$$

In a similar way, we obtain the exotic neutrinos mass matrix.

$$-\mathcal{L}_{mass}^{\nu_{4,5}} = (\bar{N}_{4R}, \bar{N}_{5R}) \begin{pmatrix} 0 & -\frac{v_d}{\sqrt{2}}Y_{\nu_5} \\ \frac{v_u}{\sqrt{2}}Y_{\nu_4} & 0 \end{pmatrix} \begin{pmatrix} N_{4L} \\ N_{5L} \end{pmatrix} + h.c. \tag{10}$$

Adopting the unitary transformations

$$\begin{pmatrix} N'_{4L} \\ N'_{5L} \end{pmatrix} = U_N^\dagger \begin{pmatrix} N_{4L} \\ N_{5L} \end{pmatrix}, \quad \begin{pmatrix} N'_{4R} \\ N'_{5R} \end{pmatrix} = W_N^\dagger \begin{pmatrix} N_{4R} \\ N_{5R} \end{pmatrix}, \tag{11}$$

the mass matrix of exotic neutrinos are diagonalized as

$$W_N^\dagger \begin{pmatrix} 0 & -\frac{v_d}{\sqrt{2}}Y_{\nu_5} \\ \frac{v_u}{\sqrt{2}}Y_{\nu_4} & 0 \end{pmatrix} U_N = \text{diag}(m_{\nu_4}, m_{\nu_5}). \tag{12}$$

The mass matrix of exotic charged lepton are shown here

$$-\mathcal{L}_{mass}^{L_{4,5}} = (\bar{L}_{4R}, \bar{L}_{5R}) \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_{e_5} \\ -\frac{v_d}{\sqrt{2}} Y_{e_4} & 0 \end{pmatrix} \begin{pmatrix} L_{4L} \\ L_{5L} \end{pmatrix} + h.c. \quad (13)$$

With the unitary transformations

$$\begin{pmatrix} L'_{4L} \\ L'_{5L} \end{pmatrix} = U_L^\dagger \cdot \begin{pmatrix} L_{4L} \\ L_{5L} \end{pmatrix}, \quad \begin{pmatrix} L'_{4R} \\ L'_{5R} \end{pmatrix} = W_L^\dagger \cdot \begin{pmatrix} L_{4R} \\ L_{5R} \end{pmatrix}, \quad (14)$$

one can diagonalize the mass matrix of exotic charged lepton as

$$W_L^\dagger \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_{e_5} \\ -\frac{v_d}{\sqrt{2}} Y_{e_4} & 0 \end{pmatrix} U_L = \text{diag}(m_{l_4}, m_{l_5}) \quad (15)$$

From the superpotential and the soft breaking terms in BLMSSM Eq.(4), the mass squared matrices of the scalar neutrinos and scalar exotic charged leptons are obtained.

$$\begin{aligned} -\mathcal{L}_S^{mass} = & \tilde{n}^\dagger \cdot \mathcal{M}_{\tilde{n}}^2 \cdot \tilde{n} + \tilde{n}_4^\dagger \cdot \mathcal{M}_{\tilde{n}_4}^2 \cdot \tilde{n}_4 + \tilde{n}_5^\dagger \cdot \mathcal{M}_{\tilde{n}_5}^2 \cdot \tilde{n}_5 \\ & + \tilde{e}_4^\dagger \cdot \mathcal{M}_{\tilde{e}_4}^2 \cdot \tilde{e}_4 + \tilde{e}_5^\dagger \cdot \mathcal{M}_{\tilde{e}_5}^2 \cdot \tilde{e}_5 \end{aligned} \quad (16)$$

with $\tilde{n}^T = (\tilde{\nu}^I, \tilde{N}^{cI*})$, $\tilde{n}_4^T = (\tilde{N}_4, \tilde{N}_4^{c*})$, $\tilde{e}_4^T = (\tilde{E}_4, \tilde{E}_4^{c*})$, $\tilde{n}_5^T = (\tilde{N}_5, \tilde{N}_5^{c*})$ and $\tilde{e}_5^T = (\tilde{E}_5, \tilde{E}_5^{c*})$.

The concrete forms for the mass squared matrices $\mathcal{M}_{\tilde{n}}$, $\mathcal{M}_{\tilde{n}_4}$, $\mathcal{M}_{\tilde{e}_4}$, $\mathcal{M}_{\tilde{n}_5}$ and $\mathcal{M}_{\tilde{e}_5}$ are collected here.

The scalar neutrinos are enlarged by the superfields \tilde{N}^c and the mass squared matrix reads as

$$\begin{aligned} \mathcal{M}_{\tilde{n}}^2(\tilde{\nu}_I^* \tilde{\nu}_J) &= \frac{g_1^2 + g_2^2}{8} (v_d^2 - v_u^2) \delta_{IJ} + g_L^2 (\bar{v}_L^2 - v_L^2) \delta_{IJ} + \frac{v_u^2}{2} (Y_\nu^\dagger Y_\nu)_{IJ} + (M_L^2)_{IJ}, \\ \mathcal{M}_{\tilde{n}}^2(\tilde{N}_I^{c*} \tilde{N}_J^c) &= -g_L^2 (\bar{v}_L^2 - v_L^2) \delta_{IJ} + \frac{v_u^2}{2} (Y_\nu^\dagger Y_\nu)_{IJ} + 2\bar{v}_L^2 (\lambda_{N_c}^\dagger \lambda_{N_c})_{IJ} \\ &\quad + (M_\nu^2)_{IJ} + \mu_L \frac{v_L}{\sqrt{2}} (\lambda_{N_c})_{IJ} - \frac{\bar{v}_L}{\sqrt{2}} (A_{N_c})_{IJ}, \\ \mathcal{M}_{\tilde{n}}^2(\tilde{\nu}_I \tilde{N}_J^c) &= \mu^* \frac{v_d}{\sqrt{2}} (Y_\nu)_{IJ} - v_u \bar{v}_L (Y_\nu^\dagger \lambda_{N_c})_{IJ} + \frac{v_u}{\sqrt{2}} (A_N)_{IJ}. \end{aligned} \quad (17)$$

The mass squared matrix of the 4th generation scalar neutrinos is

$$\begin{aligned} \mathcal{M}_{\tilde{n}_4}^2(\tilde{N}_4^* \tilde{N}_4) &= \frac{g_1^2 + g_2^2}{8} (v_d^2 - v_u^2) + g_L^2 L_4 (\bar{v}_L^2 - v_L^2) + \frac{v_u^2}{2} |Y_{\nu_4}|^2 + M_{\tilde{L}_4}^2, \\ \mathcal{M}_{\tilde{n}_4}^2(\tilde{N}_4^{c*} \tilde{N}_4^c) &= -g_L^2 L_4 (\bar{v}_L^2 - v_L^2) + \frac{v_u^2}{2} |Y_{\nu_4}|^2 + M_{\tilde{\nu}_4}^2, \\ \mathcal{M}_{\tilde{n}_4}^2(\tilde{N}_4 \tilde{N}_4^c) &= \mu^* \frac{v_d}{\sqrt{2}} Y_{\nu_4} + A_{\nu_4} \frac{v_u}{\sqrt{2}}. \end{aligned} \quad (18)$$

The mass squared matrix of the 4th generation scalar charged leptons is

$$\begin{aligned}\mathcal{M}_{\tilde{e}_4}^2(\tilde{E}_4^* \tilde{E}_4) &= \frac{g_1^2 - g_2^2}{8}(v_d^2 - v_u^2) + g_L^2 L_4(\bar{v}_L^2 - v_L^2) + \frac{v_d^2}{2}|Y_{e_4}|^2 + M_{\tilde{L}_4}^2, \\ \mathcal{M}_{\tilde{e}_4}^2(\tilde{E}_4^{c*} \tilde{E}_4^c) &= -\frac{g_1^2}{4}(v_d^2 - v_u^2) - g_L^2 L_4(\bar{v}_L^2 - v_L^2) + \frac{v_d^2}{2}|Y_{e_4}|^2 + M_{\tilde{e}_4}^2 \\ \mathcal{M}_{\tilde{e}_4}^2(\tilde{E}_4 \tilde{E}_4^c) &= \mu^* \frac{v_u}{\sqrt{2}} Y_{e_4} + A_{e_4} \frac{v_d}{\sqrt{2}}.\end{aligned}\tag{19}$$

The mass squared matrix of the 5th generation scalar neutrinos is

$$\begin{aligned}\mathcal{M}_{\tilde{n}_5}^2(\tilde{N}_5^{c*} \tilde{N}_5^c) &= -\frac{g_1^2 + g_2^2}{8}(v_d^2 - v_u^2) - g_L^2(3 + L_4)(\bar{v}_L^2 - v_L^2) + \frac{v_d^2}{2}|Y_{\nu_5}|^2 + M_{\tilde{L}_5}^2, \\ \mathcal{M}_{\tilde{n}_5}^2(\tilde{N}_5^* \tilde{N}_5) &= g_L^2(3 + L_4)(\bar{v}_L^2 - v_L^2) + \frac{v_d^2}{2}|Y_{\nu_5}|^2 + M_{\tilde{\nu}_5}^2 \\ \mathcal{M}_{\tilde{n}_5}^2(\tilde{N}_5 \tilde{N}_5^c) &= \mu^* \frac{v_u}{\sqrt{2}} Y_{\nu_5} + A_{\nu_5} \frac{v_d}{\sqrt{2}}.\end{aligned}\tag{20}$$

The mass squared matrix of the 5th generation scalar charged leptons is

$$\begin{aligned}\mathcal{M}_{\tilde{e}_5}^2(\tilde{E}_5^{c*} \tilde{E}_5^c) &= -\frac{g_1^2 - g_2^2}{8}(v_d^2 - v_u^2) - g_L^2(3 + L_4)(\bar{v}_L^2 - v_L^2) + \frac{v_u^2}{2}|Y_{e_5}|^2 + M_{\tilde{L}_5}^2, \\ \mathcal{M}_{\tilde{e}_5}^2(\tilde{E}_5^* \tilde{E}_5) &= \frac{g_1^2}{4}(v_d^2 - v_u^2) + g_L^2(3 + L_4)(\bar{v}_L^2 - v_L^2) + \frac{v_u^2}{2}|Y_{e_5}|^2 + M_{\tilde{e}_5}^2 \\ \mathcal{M}_{\tilde{e}_5}^2(\tilde{E}_5 \tilde{E}_5^c) &= \mu^* \frac{v_d}{\sqrt{2}} Y_{e_5} + A_{e_5} \frac{v_u}{\sqrt{2}}.\end{aligned}\tag{21}$$

B. The needed couplings

We deduce the couplings between the charged Higgs and the exotic leptons(4,5) from the super potential in Eq.(4).

$$\begin{aligned}\mathcal{L}_{H^\pm N' L'} &= \sum_{i,j=1}^2 \bar{N}'_{i+3} \left((Y_{e_4}^* (U_N^\dagger)^{i1} W_L^{2j} + Y_{\nu_5}^* (U_N^\dagger)^{i2} W_L^{1j}) \cos \beta \omega_+ \right. \\ &\quad \left. - (Y_{\nu_4} (W_N^\dagger)^{i2} U_L^{1j} + Y_{e_5} (W_N^\dagger)^{i1} U_L^{2j}) \sin \beta \omega_- \right) L'_{j+3} G^+ \\ &\quad + \sum_{i,j=1}^2 \bar{N}'_{i+3} \left(- (Y_{e_4}^* (U_N^\dagger)^{i1} W_L^{2j} + Y_{\nu_5}^* (U_N^\dagger)^{i2} W_L^{1j}) \sin \beta \omega_+ \right. \\ &\quad \left. - (Y_{\nu_4} (W_N^\dagger)^{i2} U_L^{1j} + Y_{e_5} (W_N^\dagger)^{i1} U_L^{2j}) \cos \beta \omega_- \right) L'_{j+3} H^+ + h.c.\end{aligned}\tag{22}$$

The couplings between neutral CP-even Higgs and the exotic leptons(4,5) are shown here.

$$\begin{aligned}\mathcal{L}_{H^0 L' L'} &= \sum_{i,j=1}^2 \left(-\frac{Y_{e_4}}{\sqrt{2}} (W_L^\dagger)^{i2} U_L^{1j} \cos \alpha + \frac{Y_{e_5}}{\sqrt{2}} (W_L^\dagger)^{i1} U_L^{2j} \sin \alpha \right) \bar{L}'_{i+3} \omega_- L'_{j+3} H^0 \\ &\quad + \sum_{i,j=1}^2 \left(-\frac{Y_{e_4}^*}{\sqrt{2}} W_L^{2j} (U_L^\dagger)^{i1} \cos \alpha + \frac{Y_{e_5}^*}{\sqrt{2}} W_L^{1j} (U_L^\dagger)^{i2} \sin \alpha \right) \bar{L}'_{i+3} \omega_+ L'_{j+3} H^0\end{aligned}$$

$$\begin{aligned}
& + \sum_{i,j=1}^2 \left(\frac{Y_{e4}}{\sqrt{2}} (W_L^\dagger)^{i2} U_L^{1j} \sin \alpha + \frac{Y_{e5}}{\sqrt{2}} (W_L^\dagger)^{i1} U_L^{2j} \cos \alpha \right) \bar{L}'_{i+3} \omega_- L'_{j+3} h^0 \\
& + \sum_{i,j=1}^2 \left(\frac{Y_{e4}^*}{\sqrt{2}} W_L^{2j} (U_L^\dagger)^{i1} \sin \alpha + \frac{Y_{e5}^*}{\sqrt{2}} W_L^{1j} (U_L^\dagger)^{i2} \cos \alpha \right) \bar{L}'_{i+3} \omega_+ L'_{j+3} h^0. \quad (23)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{H^0 N' N'} &= \sum_{i,j=1}^2 \left(-\frac{Y_{\nu 5}}{\sqrt{2}} (W_N^\dagger)^{i1} U_N^{2j} \cos \alpha + \frac{Y_{\nu 4}}{\sqrt{2}} (W_N^\dagger)^{i2} U_N^{1j} \sin \alpha \right) \bar{N}'_{i+3} \omega_- N'_{j+3} H^0 \\
& + \sum_{i,j=1}^2 \left(-\frac{Y_{\nu 5}^*}{\sqrt{2}} W_N^{1j} (U_N^\dagger)^{i2} \cos \alpha + \frac{Y_{\nu 4}^*}{\sqrt{2}} W_N^{2j} (U_N^\dagger)^{i1} \sin \alpha \right) \bar{N}'_{i+3} \omega_+ N'_{j+3} H^0 \\
& + \sum_{i,j=1}^2 \left(\frac{Y_{\nu 5}}{\sqrt{2}} (W_N^\dagger)^{i1} U_N^{2j} \sin \alpha + \frac{Y_{\nu 4}}{\sqrt{2}} (W_N^\dagger)^{i2} U_N^{1j} \cos \alpha \right) \bar{N}'_{i+3} \omega_- N'_{j+3} h^0 \\
& + \sum_{i,j=1}^2 \left(\frac{Y_{\nu 5}^*}{\sqrt{2}} W_N^{1j} (U_N^\dagger)^{i2} \sin \alpha + \frac{Y_{\nu 4}^*}{\sqrt{2}} W_N^{2j} (U_N^\dagger)^{i1} \cos \alpha \right) \bar{N}'_{i+3} \omega_+ N'_{j+3} h^0. \quad (24)
\end{aligned}$$

Using the same method, we also get the couplings between neutral CP-odd Higgs and the exotic leptons(4,5).

$$\begin{aligned}
\mathcal{L}_{A^0 L' L'} &= \sum_{i,j=1}^2 i \left(-\frac{Y_{e4}}{\sqrt{2}} (W_L^\dagger)^{i2} U_L^{1j} \cos \beta + \frac{Y_{e5}}{\sqrt{2}} (W_L^\dagger)^{i1} U_L^{2j} \sin \beta \right) \bar{L}'_{i+3} \omega_- L'_{j+3} G^0 \\
& + \sum_{i,j=1}^2 i \left(-\frac{Y_{e4}^*}{\sqrt{2}} W_L^{2j} (U_L^\dagger)^{i1} \cos \beta + \frac{Y_{e5}^*}{\sqrt{2}} W_L^{1j} (U_L^\dagger)^{i2} \sin \beta \right) \bar{L}'_{i+3} \omega_+ L'_{j+3} G^0 \\
& + \sum_{i,j=1}^2 i \left(\frac{Y_{e4}}{\sqrt{2}} (W_L^\dagger)^{i2} U_L^{1j} \sin \beta + \frac{Y_{e5}}{\sqrt{2}} (W_L^\dagger)^{i1} U_L^{2j} \cos \beta \right) \bar{L}'_{i+3} \omega_- L'_{j+3} A^0 \\
& + \sum_{i,j=1}^2 i \left(\frac{Y_{e4}^*}{\sqrt{2}} W_L^{2j} (U_L^\dagger)^{i1} \sin \beta + \frac{Y_{e5}^*}{\sqrt{2}} W_L^{1j} (U_L^\dagger)^{i2} \cos \beta \right) \bar{L}'_{i+3} \omega_+ L'_{j+3} A^0. \\
\mathcal{L}_{A^0 N' N'} &= \sum_{i,j=1}^2 i \left(-\frac{Y_{\nu 5}}{\sqrt{2}} (W_N^\dagger)^{i1} U_N^{2j} \cos \beta + \frac{Y_{\nu 4}}{\sqrt{2}} (W_N^\dagger)^{i2} U_N^{1j} \sin \beta \right) \bar{N}'_{i+3} \omega_- N'_{j+3} G^0 \\
& + \sum_{i,j=1}^2 i \left(-\frac{Y_{\nu 5}^*}{\sqrt{2}} W_N^{1j} (U_N^\dagger)^{i2} \cos \beta + \frac{Y_{\nu 4}^*}{\sqrt{2}} W_N^{2j} (U_N^\dagger)^{i1} \sin \beta \right) \bar{N}'_{i+3} \omega_+ N'_{j+3} G^0 \\
& + \sum_{i,j=1}^2 i \left(\frac{Y_{\nu 5}}{\sqrt{2}} (W_N^\dagger)^{i1} U_N^{2j} \sin \beta + \frac{Y_{\nu 4}}{\sqrt{2}} (W_N^\dagger)^{i2} U_N^{1j} \cos \beta \right) \bar{N}'_{i+3} \omega_- N'_{j+3} A^0 \\
& + \sum_{i,j=1}^2 i \left(\frac{Y_{\nu 5}^*}{\sqrt{2}} W_N^{1j} (U_N^\dagger)^{i2} \sin \beta + \frac{Y_{\nu 4}^*}{\sqrt{2}} W_N^{2j} (U_N^\dagger)^{i1} \cos \beta \right) \bar{N}'_{i+3} \omega_+ N'_{j+3} A^0. \quad (25)
\end{aligned}$$

In the Barr-Zee type two-loop diagrams, the couplings between one vector boson and exotic leptons(4,5) are necessary.

$$\mathcal{L}_{V L' L'} = \sum_{i,j=1}^2 \left[e F_\mu \bar{L}'_{i+3} \gamma^\mu L'_{i+3} - \frac{e}{2s_W c_W} Z_\mu \bar{N}'_{i+3} \left((U_N^\dagger)^{i1} U_N^{1j} \gamma^\mu \omega_- + (W_N^\dagger)^{i1} W_N^{1j} \gamma^\mu \omega_+ \right) N'_{j+3} \right]$$

$$\begin{aligned}
& +eZ_\mu \bar{L}_{i+3} \left(\left(-\frac{s_W}{c_W} \delta_{ij} + \frac{1}{2s_W c_W} (U_L^\dagger)^{i1} U_L^{1j} \right) \gamma^\mu \omega_- + \left(-\frac{s_W}{c_W} \delta_{ij} + \frac{1}{2s_W c_W} (W_L^\dagger)^{i1} W_L^{1j} \right) \gamma^\mu \omega_+ \right) L'_{j+3} \\
& - \frac{e}{\sqrt{2}s_W} \bar{N}'_{i+3} \left((U_N^\dagger)^{i1} U_L^{1j} \gamma^\mu \omega_- - (W_N^\dagger)^{i1} W_L^{1j} \gamma^\mu \omega_+ \right) L'_{j+3} W_\mu^+ \Big] + h.c. \tag{26}
\end{aligned}$$

Here, we adopt the abbreviation notations $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, where θ_W is the Weinberg angle. The exotic scalar leptons(4,5) have contributions to muon MDM at two-loop level. The couplings of one vector boson and exotic scalar leptons(4,5) are given out.

$$\begin{aligned}
\mathcal{L}_{V\bar{L}'\bar{L}'} = & eF_\mu \sum_{i,j=1}^2 \tilde{E}_4^{i*} i\tilde{\partial}^\mu \tilde{E}_4'^j \delta^{ij} + eZ_\mu \sum_{i,j=1}^2 \left[-\frac{s_W}{c_W} \delta^{ij} + \frac{1}{2s_W c_W} (Z_{\tilde{e}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} \right] \tilde{E}_4^{i*} i\tilde{\partial}^\mu \tilde{E}_4'^j \\
& + eF_\mu \sum_{i,j=1}^2 \tilde{E}_5^{i*} i\tilde{\partial}^\mu \tilde{E}_5'^j \delta^{ij} + eZ_\mu \sum_{i,j=1}^2 \left[-\frac{s_W}{c_W} \delta^{ij} + \frac{1}{2s_W c_W} (Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} \right] \tilde{E}_5^{i*} i\tilde{\partial}^\mu \tilde{E}_5'^j \\
& - \frac{e}{2s_W c_W} Z_\mu \sum_{i,j=1}^2 (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{\nu}_4}^{1j} \tilde{N}_4^{i*} i\tilde{\partial}^\mu \tilde{N}_4'^j - \frac{e}{2s_W c_W} Z_\mu \sum_{i,j=1}^2 (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{2j} \tilde{N}_5^{i*} i\tilde{\partial}^\mu \tilde{N}_5'^j \\
& - \frac{eW_\mu^+}{\sqrt{2}s_W} \sum_{i,j=1}^2 (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} \tilde{N}_4^{i*} i\tilde{\partial}^\mu \tilde{E}_4'^j + \frac{eW_\mu^+}{\sqrt{2}s_W} \sum_{i,j=1}^2 (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} \tilde{N}_5^{i*} i\tilde{\partial}^\mu \tilde{E}_5'^j + h.c. \tag{27}
\end{aligned}$$

with $\tilde{\partial}^\mu = \vec{\partial}^\mu - \overleftarrow{\partial}^\mu$. $Z_{\tilde{\nu}_4}, Z_{\tilde{e}_4}, Z_{\tilde{\nu}_5}, Z_{\tilde{e}_5}$ are the unitary matrices to diagonalize the mass squared matrices $\mathcal{M}_{\tilde{n}_4}^2, \mathcal{M}_{\tilde{e}_4}^2, \mathcal{M}_{\tilde{n}_5}^2, \mathcal{M}_{\tilde{e}_5}^2$ respectively.

$$\begin{aligned}
Z_{\tilde{\nu}_4}^\dagger \mathcal{M}_{\tilde{n}_4}^2 Z_{\tilde{\nu}_4} &= diag(m_{\tilde{\nu}_4^1}^2, m_{\tilde{\nu}_4^2}^2), & Z_{\tilde{e}_4}^\dagger \mathcal{M}_{\tilde{e}_4}^2 Z_{\tilde{e}_4} &= diag(m_{\tilde{e}_4^1}^2, m_{\tilde{e}_4^2}^2), \\
Z_{\tilde{\nu}_5}^\dagger \mathcal{M}_{\tilde{n}_5}^2 Z_{\tilde{\nu}_5} &= diag(m_{\tilde{\nu}_5^1}^2, m_{\tilde{\nu}_5^2}^2), & Z_{\tilde{e}_5}^\dagger \mathcal{M}_{\tilde{e}_5}^2 Z_{\tilde{e}_5} &= diag(m_{\tilde{e}_5^1}^2, m_{\tilde{e}_5^2}^2). \tag{28}
\end{aligned}$$

For the couplings between vector Bosons and scalars, the VVSS type must be considered. Here, we just show the used coupling between $\gamma - V$ and two exotic scalar leptons(4,5).

$$\begin{aligned}
\mathcal{L}_{\gamma V\bar{L}'\bar{L}'} = & \frac{e^2}{\sqrt{2}s_W} F^\mu W_\mu^+ \sum_{i,j=1}^2 \left(- (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} \tilde{N}_4^{i*} \tilde{E}_4'^j + (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} \tilde{N}_5^{i*} \tilde{E}_5'^j \right) \\
& + \frac{e^2}{s_W c_W} F^\mu Z_\mu \sum_{i,j=1}^2 \left(((Z_{\tilde{e}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} - 2s_W^2 \delta_{ij}) \tilde{E}_4^{i*} \tilde{E}_4'^j + ((Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} - 2s_W^2 \delta_{ij}) \tilde{E}_5^{i*} \tilde{E}_5'^j \right) \\
& + e^2 F^\mu F_\mu \sum_{i,j=1}^2 \delta_{ij} (\tilde{E}_4^{i*} \tilde{E}_4'^j + \tilde{E}_5^{i*} \tilde{E}_5'^j) + h.c. \tag{29}
\end{aligned}$$

The couplings between charged Higgs and exotic scalar leptons(4,5) are

$$\begin{aligned}
\mathcal{L}_{H^\pm \bar{L}'\bar{L}'} = & \sum_{i,j=1}^2 \tilde{N}_4^{i*} \tilde{E}_4'^j G^+ [(L_4^u)_{ij} \sin \beta + (L_4^d)_{ij} \cos \beta] \\
& + \sum_{i,j=1}^2 \tilde{N}_4^{i*} \tilde{E}_4'^j H^+ [(L_4^u)_{ij} \cos \beta - (L_4^d)_{ij} \sin \beta]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i,j=1}^2 \tilde{N}_5^{i*} \tilde{E}_5'^{ij} G^+ [(L_5^u)_{ij} \sin \beta + (L_5^d)_{ij} \cos \beta] \\
& + \sum_{i,j=1}^2 \tilde{N}_5^{i*} \tilde{E}_5'^{ij} H^+ [(L_5^u)_{ij} \cos \beta - (L_5^d)_{ij} \sin \beta] + h.c.
\end{aligned} \tag{30}$$

with

$$\begin{aligned}
(L_4^u)_{ij} &= \frac{V_{EW} \sin \beta}{\sqrt{2}} \left(-\frac{e^2}{2s_W^2} + |Y_{\nu_4}|^2 \right) (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} - \mu Y_{e_4}^* (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{2j} \\
&\quad - \frac{V_{EW} \cos \beta}{\sqrt{2}} Y_{e_4}^* Y_{\nu_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{2j} + A_{N_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{1j}, \\
(L_4^d)_{ij} &= \frac{V_{EW} \cos \beta}{\sqrt{2}} \left(-\frac{e^2}{2s_W^2} + |Y_{e_4}|^2 \right) (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} - \mu^* Y_{\nu_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{1j} \\
&\quad - \frac{V_{EW} \sin \beta}{\sqrt{2}} Y_{e_4}^* Y_{\nu_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{2j} + A_{e_4}^* (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{2j}, \\
(L_5^u)_{ij} &= \frac{V_{EW} \sin \beta}{\sqrt{2}} \left(-\frac{e^2}{2s_W^2} + |Y_{e_5}|^2 \right) (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} - \mu Y_{\nu_5}^* (Z_{\tilde{\nu}_5}^\dagger)^{i1} Z_{\tilde{e}_5}^{2j} \\
&\quad - \frac{V_{EW} \cos \beta}{\sqrt{2}} Y_{e_5} Y_{\nu_5}^* (Z_{\tilde{\nu}_5}^\dagger)^{i1} Z_{\tilde{e}_5}^{1j} + A_{e_5} (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{1j}, \\
(L_5^d)_{ij} &= \frac{V_{EW} \cos \beta}{\sqrt{2}} \left(-\frac{e^2}{2s_W^2} + |Y_{e_5}|^2 \right) (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} - \mu^* Y_{e_5} (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{1j} \\
&\quad - \frac{V_{EW} \sin \beta}{\sqrt{2}} Y_{e_5} Y_{\nu_5}^* (Z_{\tilde{\nu}_5}^\dagger)^{i1} Z_{\tilde{e}_5}^{1j} + A_{N_5}^* (Z_{\tilde{\nu}_5}^\dagger)^{i1} Z_{\tilde{e}_5}^{2j}.
\end{aligned} \tag{31}$$

The couplings between the neutral CP-even Higgs and the exotic scalar lepton(4,5) are also collected here.

$$\begin{aligned}
\mathcal{L}_{H^0 \tilde{L}' \tilde{L}'} &= \sum_{i,j=1}^2 \tilde{N}_4^{i*} \tilde{N}_4'^{ij} \left(H^0 [(N_4^u)_{ij} \sin \alpha + (N_4^d)_{ij} \cos \alpha] + h^0 [(N_4^u)_{ij} \cos \alpha - (N_4^d)_{ij} \sin \alpha] \right) \\
&+ \sum_{i,j=1}^2 \tilde{N}_5^{i*} \tilde{N}_5'^{ij} \left(H^0 [(N_5^u)_{ij} \sin \alpha + (N_5^d)_{ij} \cos \alpha] + h^0 [(N_5^u)_{ij} \cos \alpha - (N_5^d)_{ij} \sin \alpha] \right) \\
&+ \sum_{i,j=1}^2 \tilde{E}_4^{i*} \tilde{E}_4'^{ij} \left(H^0 [(E_4^u)_{ij} \sin \alpha + (E_4^d)_{ij} \cos \alpha] + h^0 [(E_4^u)_{ij} \cos \alpha - (E_4^d)_{ij} \sin \alpha] \right) \\
&+ \sum_{i,j=1}^2 \tilde{E}_5^{i*} \tilde{E}_5'^{ij} \left(H^0 [(E_5^u)_{ij} \sin \alpha + (E_5^d)_{ij} \cos \alpha] + h^0 [(E_5^u)_{ij} \cos \alpha - (E_5^d)_{ij} \sin \alpha] \right) \tag{32}
\end{aligned}$$

where the concrete forms of the coupling constants $N_{4,5}^{u,d}, E_{4,5}^{u,d}$ are

$$\begin{aligned}
(N_4^u)_{ij} &= \frac{e^2}{4s_W^2 c_W^2} V_{EW} \sin \beta (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{\nu}_4}^{1j} - V_{EW} \sin \beta |Y_{\nu_4}|^2 \delta_{ij} - \frac{A_{N_4}}{\sqrt{2}} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{\nu}_4}^{1j}, \\
(N_4^d)_{ij} &= -\frac{e^2}{4s_W^2 c_W^2} V_{EW} \cos \beta (Z_{\tilde{\nu}_4}^\dagger)^{i1} Z_{\tilde{\nu}_4}^{1j} - \frac{\mu^*}{\sqrt{2}} Y_{\nu_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{\nu}_4}^{1j},
\end{aligned}$$

$$\begin{aligned}
(N_5^u)_{ij} &= -\frac{e^2}{4s_W^2 c_W^2} V_{EW} \sin \beta (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{2j} - \frac{\mu^*}{\sqrt{2}} Y_{\nu_5} (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{1j}, \\
(N_5^d)_{ij} &= \frac{e^2}{4s_W^2 c_W^2} V_{EW} \cos \beta (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{2j} - V_{EW} \cos \beta |Y_{\nu_5}|^2 \delta_{ij} - \frac{A_{N_5}}{\sqrt{2}} (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{1j},
\end{aligned} \tag{33}$$

$$\begin{aligned}
(E_4^u)_{ij} &= -e^2 V_{EW} \sin \beta \left(\frac{1}{2c_W^2} \delta_{ij} + \frac{1-4s_W^2}{4s_W^2 c_W^2} (Z_{\tilde{e}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} \right) - \frac{\mu^*}{\sqrt{2}} Y_{e_4} (Z_{\tilde{e}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{1j}, \\
(E_4^d)_{ij} &= e^2 V_{EW} \cos \beta \left(\frac{1}{2c_W^2} \delta_{ij} + \frac{1-4s_W^2}{4s_W^2 c_W^2} (Z_{\tilde{e}_4}^\dagger)^{i1} Z_{\tilde{e}_4}^{1j} \right) - V_{EW} \cos \beta |Y_{e_4}|^2 \delta_{ij} - \frac{A_{E_4}}{\sqrt{2}} (Z_{\tilde{e}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{1j}, \\
(E_5^u)_{ij} &= e^2 V_{EW} \sin \beta \left(\frac{1}{2c_W^2} \delta_{ij} + \frac{1-4s_W^2}{4s_W^2 c_W^2} (Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{2j} \right) - V_{EW} \sin \beta |Y_{e_5}|^2 \delta_{ij} - \frac{A_{E_5}}{\sqrt{2}} (Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{1j}, \\
(E_5^d)_{ij} &= -e^2 V_{EW} \cos \beta \left(\frac{1}{2c_W^2} \delta_{ij} + \frac{1-4s_W^2}{4s_W^2 c_W^2} (Z_{\tilde{e}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{2j} \right) - \frac{\mu^*}{\sqrt{2}} Y_{e_5} (Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{1j}.
\end{aligned} \tag{34}$$

Similarly, the couplings between the CP-odd Higgs and exotic scalar leptons(4,5) are obtained.

$$\begin{aligned}
\mathcal{L}_{A^0 \tilde{L}' \tilde{L}'} &= -\frac{i}{\sqrt{2}} \sum_{i,j=1}^2 \tilde{N}_4^{i*} \tilde{N}_4'^{ij} (\cos \beta A^0 + \sin \beta G^0) A_{N_4} (Z_{\tilde{\nu}_4}^\dagger)^{i2} Z_{\tilde{\nu}_4}^{1j} \\
&\quad - \frac{i}{\sqrt{2}} \sum_{i,j=1}^2 \tilde{N}_5^{i*} \tilde{N}_5'^{ij} (-\sin \beta A^0 + \cos \beta G^0) A_{N_5} (Z_{\tilde{\nu}_5}^\dagger)^{i2} Z_{\tilde{\nu}_5}^{1j} \\
&\quad - \frac{i}{\sqrt{2}} \sum_{i,j=1}^2 \tilde{E}_4^{i*} \tilde{E}_4'^{ij} (-\sin \beta A^0 + \cos \beta G^0) A_{E_4} (Z_{\tilde{e}_4}^\dagger)^{i2} Z_{\tilde{e}_4}^{1j} \\
&\quad - \frac{i}{\sqrt{2}} \sum_{i,j=1}^2 \tilde{E}_5^{i*} \tilde{E}_5'^{ij} (\cos \beta A^0 + \sin \beta G^0) A_{E_5} (Z_{\tilde{e}_5}^\dagger)^{i2} Z_{\tilde{e}_5}^{1j}.
\end{aligned} \tag{35}$$

The couplings between neutral Higgs and exotic quarks (scalar quarks) can be found in our previous work[8]. In Ref.[9], the couplings between charged Higgs and exotic quarks are also given out. To complete the couplings, we deduce the charged Higgs-exotic scalar quarks couplings.

$$\begin{aligned}
\mathcal{L}_{H^\pm \tilde{Q} \tilde{Q}} &= \sum_{j,k=1}^4 \tilde{U}_j^* \tilde{\mathcal{D}}_k G^+ [(R^u)_{jk} \sin \beta + (R^d)_{jk} \cos \beta] \\
&\quad + \sum_{j,k=1}^4 \tilde{U}_j^* \tilde{\mathcal{D}}_k H^+ [(R^u)_{jk} \cos \beta - (R^d)_{jk} \sin \beta] + h.c.
\end{aligned} \tag{36}$$

The concrete forms of the coupling constants $(R^u)_{jk}, (R^d)_{jk}$ read as

$$\begin{aligned}
(R^u)_{jk} &= -\frac{\sqrt{2}e^2}{4s_W^2} v_u (U_{j1}^\dagger D_{1k} + U_{j3}^\dagger D_{3k}) - \mu Y_{d_4}^* U_{j1}^\dagger D_{2k} - \mu Y_{u_5}^* U_{j4}^\dagger D_{3k} \\
&\quad + \frac{v_B}{\sqrt{2}} \lambda_Q^* Y_{u_4} U_{j2}^\dagger D_{3k} + \frac{v_d}{\sqrt{2}} Y_{u_4} Y_{d_4}^* U_{j2}^\dagger D_{2k} + \frac{v_d}{\sqrt{2}} Y_{d_5} Y_{u_5}^* U_{j4}^\dagger D_{4k}
\end{aligned}$$

$$\begin{aligned}
& -\frac{v_B}{\sqrt{2}}Y_{d_5}\lambda_Q^*U_{j1}^\dagger D_{4k} - \frac{\bar{v}_B}{\sqrt{2}}Y_{u_4}\lambda_U^*U_{j4}^\dagger D_{1k} - \frac{\bar{v}_B}{\sqrt{2}}Y_{d_5}\lambda_d^*U_{j2}^\dagger D_{2k} \\
& + A_{u_4}U_{j2}^\dagger D_{1k} + A_{d_5}U_{j3}^\dagger D_{4k}, \\
(R^d)_{jk} = & -\frac{\sqrt{2}e^2}{4s_W^2}v_d(U_{j1}^\dagger D_{1k} + U_{j3}^\dagger D_{3k}) - \mu^*Y_{u_4}U_{j2}^\dagger D_{1k} - \mu^*Y_{d_5}U_{j3}^\dagger D_{4k} \\
& -\frac{v_B}{\sqrt{2}}\lambda_QY_{d_4}^*U_{j3}^\dagger D_{2k} + \frac{v_u}{\sqrt{2}}Y_{u_4}Y_{d_4}^*U_{j2}^\dagger D_{2k} + \frac{v_B}{\sqrt{2}}\lambda_QY_{u_5}^*U_{j4}^\dagger D_{1k} \\
& + \frac{v_u}{\sqrt{2}}Y_{d_5}Y_{u_5}^*U_{j4}^\dagger D_{4k} - \frac{\bar{v}_B}{\sqrt{2}}Y_{d_4}^*\lambda_DU_{j1}^\dagger D_{4k} - \frac{\bar{v}_B}{\sqrt{2}}Y_{u_5}^*\lambda_UU_{j2}^\dagger D_{3k} \\
& + A_{d_4}^*U_{j1}^\dagger D_{2k} + A_{u_5}^*U_{j4}^\dagger D_{3k}.
\end{aligned} \tag{37}$$

One vector Boson can couple with the exotic scalar quarks

$$\begin{aligned}
\mathcal{L}_{V\tilde{Q}\tilde{Q}} = & \frac{-e}{\sqrt{2}s_W} \sum_{j,\beta=1}^4 (U_{j1}^\dagger D_{1\beta} - U_{j3}^\dagger D_{3\beta}) W_\mu^+ \tilde{\mathcal{U}}_j^* i\tilde{\partial}^\mu \tilde{\mathcal{D}}_\beta - \frac{2}{3}e \sum_{j,\beta=1}^4 \delta_{j\beta} F_\mu \tilde{\mathcal{U}}_j^* i\tilde{\partial}^\mu \tilde{\mathcal{U}}_\beta \\
& + \frac{e}{3} \sum_{j,\beta=1}^4 \delta_{j\beta} F_\mu \tilde{\mathcal{D}}_j^* i\tilde{\partial}^\mu \tilde{\mathcal{D}}_\beta + \frac{e}{6s_W c_W} \sum_{j,\beta=1}^4 (4s_W^2 \delta_{j\beta} - 3(U_{j1}^\dagger U_{1\beta} + U_{j3}^\dagger U_{3\beta})) Z_\mu \tilde{\mathcal{U}}_j^* i\tilde{\partial}^\mu \tilde{\mathcal{U}}_\beta \\
& + \frac{e}{6s_W c_W} \sum_{j,\beta=1}^4 (-2s_W^2 \delta_{j\beta} + 3(D_{j1}^\dagger D_{1\beta} + D_{j3}^\dagger D_{3\beta})) Z_\mu \tilde{\mathcal{D}}_j^* i\tilde{\partial}^\mu \tilde{\mathcal{D}}_\beta + h.c.
\end{aligned} \tag{38}$$

The couplings between photon-vector boson-exotic scalar quarks must be taken into account.

$$\begin{aligned}
\mathcal{L}_{\gamma V\tilde{Q}\tilde{Q}} = & \frac{e^2}{3\sqrt{2}s_W} \sum_{i,j}^4 (U_{i1}^\dagger D_{1j} - U_{i3}^\dagger D_{3j}) W_\mu^+ F^\mu \tilde{\mathcal{U}}_i^* \tilde{\mathcal{D}}_j + \frac{4e^2}{9} \sum_{i,j}^4 \delta_{ij} F_\mu F^\mu \tilde{\mathcal{U}}_i^* \tilde{\mathcal{U}}_j \\
& + \frac{e^2}{9} \sum_{i,j}^4 \delta_{ij} F_\mu F^\mu \tilde{\mathcal{D}}_i^* \tilde{\mathcal{D}}_j + \frac{e^2}{9s_W c_W} \sum_{i,j=1}^4 (6(U_{i1}^\dagger U_{1j} + U_{i3}^\dagger U_{3j}) - 8s_W^2 \delta_{ij}) Z^\mu F_\mu \tilde{\mathcal{U}}_i^* \tilde{\mathcal{U}}_j \\
& + \frac{e^2}{9s_W c_W} \sum_{i,j=1}^4 (3(D_{i1}^\dagger D_{1j} + D_{i3}^\dagger D_{3j}) - 2s_W^2 \delta_{ij}) Z^\mu F_\mu \tilde{\mathcal{D}}_i^* \tilde{\mathcal{D}}_j + h.c.
\end{aligned} \tag{39}$$

Because the exotic quark are very heavy, they can give considerable contribution to the muon MDM through the coupling between Higgs and exotic quarks. We give out the coupling between vector Boson and exotic quarks.

$$\begin{aligned}
\mathcal{L}_{VQ\bar{Q}} = & -\frac{2e}{3}F_\mu \sum_{i=1}^2 \bar{t}_{i+3} \gamma^\mu t_{i+3} + \frac{e}{3}F_\mu \sum_{i=1}^2 \bar{b}_{i+3} \gamma^\mu b_{i+3} \\
& + \frac{e}{6s_W c_W} Z_\mu \sum_{j,k=1}^2 \bar{t}_{j+3} \left[\left((1 - 4c_W^2) \delta_{jk} + 3(U_t^\dagger)_{j2} (U_t)_{2k} \right) \gamma^\mu \omega_- \right. \\
& \left. + \left((1 - 4c_W^2) \delta_{jk} + 3(W_t^\dagger)_{j2} (W_t)_{2k} \right) \gamma^\mu \omega_+ \right] t_{k+3} \\
& + \frac{e}{6s_W c_W} Z_\mu \sum_{j,k=1}^2 \bar{b}_{j+3} \left[\left((1 + 2c_W^2) \delta_{jk} - 3(U_b^\dagger)_{j2} (U_b)_{2k} \right) \gamma^\mu \omega_- \right.
\end{aligned}$$

$$\begin{aligned}
& + \left((1 + 2c_W^2) \delta_{jk} - 3(W_b^\dagger)_{j2} (W_b)_{2k} \right) \gamma^\mu \omega_+ \Big] b_{k+3} \\
& + \frac{eW_\mu^+}{\sqrt{2}s_W} \sum_{j,k=1}^2 \bar{t}_{j+3} \left[(W_t^\dagger)_{j1} (W_b)_{1k} \gamma^\mu \omega_+ - (U_t^\dagger)_{j1} (U_b)_{1k} \gamma^\mu \omega_- \right] b_{k+3} + h.c. \quad (40)
\end{aligned}$$

III. FORMULATION

We use the effective Lagrangian method, the Feynman amplitude can be expressed by these dimension 6 operators.

$$\begin{aligned}
\mathcal{O}_1^\mp &= \frac{1}{(4\pi)^2} \bar{l} (i\mathcal{D})^3 \omega_\mp l, \\
\mathcal{O}_2^\mp &= \frac{eQ_f}{(4\pi)^2} \overline{(i\mathcal{D}_\mu l)} \gamma^\mu F \cdot \sigma \omega_\mp l, \\
\mathcal{O}_3^\mp &= \frac{eQ_f}{(4\pi)^2} \bar{l} F \cdot \sigma \gamma^\mu \omega_\mp (i\mathcal{D}_\mu l), \\
\mathcal{O}_4^\mp &= \frac{eQ_f}{(4\pi)^2} \bar{l} (\partial^\mu F_{\mu\nu}) \gamma^\nu \omega_\mp l, \\
\mathcal{O}_5^\mp &= \frac{m_l}{(4\pi)^2} \bar{l} (i\mathcal{D})^2 \omega_\mp l, \\
\mathcal{O}_6^\mp &= \frac{eQ_f m_l}{(4\pi)^2} \bar{l} F \cdot \sigma \omega_\mp l. \quad (41)
\end{aligned}$$

with $\mathcal{D}_\mu = \partial_\mu + ieA_\mu$ and $\omega_\mp = \frac{1 \mp \gamma_5}{2}$. $F_{\mu\nu}$ is the electromagnetic field strength, and m_l is the lepton mass. Using the equations of motion to the incoming and out going leptons separately, only the $\mathcal{O}_{2,3,6}^\mp$ contribute to lepton MDM and EDM. Therefore, the Wilson coefficients of the operators $\mathcal{O}_{2,3,6}^\mp$ in the effective Lagrangian are of interest and their dimensions are -2. The lepton MDM is the combination of the Wilson coefficients $C_{2,3,6}^\mp$ and can be obtained from the following effective Lagrangian

$$\mathcal{L}_{MDM} = \frac{e}{4m_l} a_l \bar{l} \sigma^{\mu\nu} l F_{\mu\nu}. \quad (42)$$

A. the one-loop corrections

In BLMSSM, the masses of the neutrinos, scalar neutrinos and scalar charged leptons are all adopted comparing with those in MSSM. In BLMSSM, λ_B (the superpartners of the new baryon boson) and $\psi_{\Phi_B}, \psi_{\varphi_B}$ (the superpartners of the $SU(2)_L$ singlets Φ_B, φ_B) mix and generate three baryon neutralinos. Three lepton neutralinos are made up of λ_L (the superpartners of the new lepton boson) and $\psi_{\Phi_L}, \psi_{\varphi_L}$ (the superpartners of the $SU(2)_L$

singlets Φ_L, φ_L). There are also four MSSM neutralinos and they do not mix with baryon neutralinos and lepton neutralinos. That is to say the four MSSM neutralinos in BLMSSM are same as those in MSSM. Therefore in BLMSSM there are ten neutralinos, but three baryon neutralinos and three lepton neutralinos have none contribution to lepton MDM in our studied diagrams.

The one loop new physics contributions to muon MDM, comes from the diagrams in Fig.1. The one-loop triangle diagrams are obtained from the one-loop self-energy diagrams by attaching a photon on the internal line in all possible ways. In BLMSSM, the one-loop corrections are similar to the MSSM results in analytic form. The differences are: 1. The squared mass matrixes of scalar leptons because of new parameters g_L, \bar{v}_L, v_L and so on. 2. Right-handed neutrinos and scalar neutrinos are introduced, which leads to the neutrinos and scalar neutrinos are doubled. The one-loop self-energy diagrams can be divided into four parts according to the virtual particles: 1. scalar neutrino-chargino; 2. neutral Higgs and lepton; 3. charged Higgs and neutrino; 4. scalar charged lepton and neutralino. The

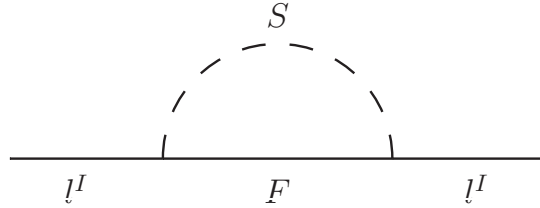


FIG. 1: The generic one loop self-energy diagram for lepton.

lepton flavor mixing is also taken into account, whose contribution is considerable. The corrections to muon MDM from neutralinos and scalar leptons are expressed as

$$a_1^{\tilde{L}\chi^0} = -\frac{e^2}{2s_W^2} \sum_{i=1}^6 \sum_{j=1}^4 \left[\text{Re}[(\mathcal{S}_1)_{ij}^I (\mathcal{S}_2)_{ij}^{I*}] \sqrt{x_{\chi_j^0} x_{m_{l^I}} x_{\tilde{L}_i}} \frac{\partial^2 \mathcal{B}(x_{\chi_j^0}, x_{\tilde{L}_i})}{\partial x_{\tilde{L}_i}^2} \right. \\ \left. + \frac{1}{3} (|(\mathcal{S}_1)_{ij}^I|^2 + |(\mathcal{S}_2)_{ij}^I|^2) x_{\tilde{L}_i} x_{m_{l^I}} \frac{\partial \mathcal{B}_1(x_{\chi_j^0}, x_{\tilde{L}_i})}{\partial x_{\tilde{L}_i}} \right], \quad (43)$$

where the couplings $(\mathcal{S}_1)_{ij}^I, (\mathcal{S}_2)_{ij}^I$ are shown as

$$(\mathcal{S}_1)_{ij}^I = \frac{1}{c_W} Z_{\tilde{L}}^{I*} (Z_N^{1j} s_W + Z_N^{2j} c_W) - \frac{m_{l^I}}{\cos \beta m_W} Z_{\tilde{L}}^{(I+3)*} Z_N^{3j}, \\ (\mathcal{S}_2)_{ij}^I = -2 \frac{s_W}{c_W} Z_{\tilde{L}}^{(I+3)*} Z_N^{1j*} - \frac{m_{l^I}}{\cos \beta m_W} Z_{\tilde{L}}^{I*} Z_N^{3j*}. \quad (44)$$

The matrices $Z_{\tilde{L}}, Z_N$ respectively diagonalize the mass matrices of scalar lepton and neutralino. The concrete forms of the functions $\mathcal{B}(x, y)$, $\mathcal{B}_1(x, y)$ are

$$\mathcal{B}(x, y) = \frac{1}{16\pi^2} \left(\frac{x \ln x}{y - x} + \frac{y \ln y}{x - y} \right), \quad \mathcal{B}_1(x, y) = \left(\frac{\partial}{\partial y} + \frac{y}{2} \frac{\partial^2}{\partial y^2} \right) \mathcal{B}(x, y). \quad (45)$$

In a similar way, the corrections from chargino and scalar neutrino are also obtained.

$$\begin{aligned} a_1^{\tilde{\nu}\chi^\pm} = & \sum_{J=1}^3 \sum_{i,j=1}^2 \frac{e^2}{s_W^2} \left[\sqrt{2} \frac{m_{lI}}{m_W} \text{Re}[Z_+^{1j} Z_-^{2j}] |Z_{\tilde{\nu}IJ}^{1i}|^2 \sqrt{x_{\chi_j^\pm} x_{lI}} \mathcal{B}_1(x_{\tilde{\nu}Ji}, x_{\chi_j^\pm}) \right. \\ & \left. + \frac{1}{3} (|Z_+^{1j} Z_{\tilde{\nu}IJ}^{1i*}|^2 + \frac{m_{lI}^2}{2m_W^2} |Z_-^{2j*} Z_{\tilde{\nu}IJ}^{1i*}|^2) x_{\chi_j^\pm} x_{lI} \frac{\partial \mathcal{B}_1(x_{\tilde{\nu}Ji}, x_{\chi_j^\pm})}{\partial x_{\chi_j^\pm}} \right]. \end{aligned} \quad (46)$$

Here, Z_-, Z_+ are used to diagonalize the chargino mass matrix. Because the right-hand neutrino are introduced in BLMSSM, their super partners lead to six scalar neutrinos. The mass squared matrix of scalar neutrino are diagonalized by $Z_{\tilde{\nu}IJ}$.

Though the Higgs contributions to muon MDM are suppressed by the factor $\frac{m_{lI}^2}{m_W^2}$, we show their results here. The one loop Higgs contributions to muon MDM are small. Firstly, we show the analytic results from the neutral Higgs.

$$\begin{aligned} a_1^{H^0l} = & -\frac{e^2 m_{lI}^2}{2s_W^2 m_W^2} \left[\cos^2 \alpha \left(x_{lI} \mathcal{B}_1(x_{H^0}, x_{lI}) - \frac{1}{3} x_{lI}^2 \frac{\partial}{\partial x_{lI}} \mathcal{B}_1(x_{H^0}, x_{lI}) \right) \right. \\ & + \sin^2 \alpha \left(x_{lI} \mathcal{B}_1(x_{h^0}, x_{lI}) - \frac{1}{3} x_{lI}^2 \frac{\partial}{\partial x_{lI}} \mathcal{B}_1(x_{h^0}, x_{lI}) \right) \\ & + \cos^2 \beta \left(x_{lI} \mathcal{B}_1(x_{G^0}, x_{F_1}) + \frac{1}{3} x_{lI}^2 \frac{\partial}{\partial x_{lI}} \mathcal{B}_1(x_{G^0}, x_{lI}) \right) \\ & \left. + \sin^2 \beta \left(x_{lI} \mathcal{B}_1(x_{A^0}, x_{F_1}) + \frac{1}{3} x_{lI}^2 \frac{\partial}{\partial x_{lI}} \mathcal{B}_1(x_{A^0}, x_{lI}) \right) \right] \end{aligned} \quad (47)$$

The charged Higgs contributions are written as

$$\begin{aligned} a_1^{H^\pm \nu} = & \sum_{J=1}^3 \sum_{i=1}^2 \left[-\frac{1}{3} (|Y_{\nu J} \cos \beta W_{\nu IJ}^{2i*}|^2 + |Y_{lI}^* \sin \beta U_{\nu IJ}^{1i*}|^2) x_{lI} x_{H^\pm} \frac{\partial \mathcal{B}_1(x_{\nu Ji}, x_{H^\pm})}{\partial x_{H^\pm}} \right. \\ & + \frac{\sin 2\beta}{2} \text{Re}[Y_{\nu J} Y_{lI} W_{\nu IJ}^{2i*} U_{\nu IJ}^{1i}] \sqrt{x_{\nu Ji} x_{lI}} \left(x_{G^\pm} \frac{\partial^2 \mathcal{B}(x_{\nu Ji}, x_{G^\pm})}{\partial x_{G^\pm}^2} - x_{H^\pm} \frac{\partial^2 \mathcal{B}(x_{\nu Ji}, x_{H^\pm})}{\partial x_{H^\pm}^2} \right) \\ & \left. - \frac{1}{3} (|Y_{\nu J} \sin \beta W_{\nu IJ}^{2i*}|^2 + |Y_{lI}^* \cos \beta U_{\nu IJ}^{1i*}|^2) x_{lI} x_{G^\pm} \frac{\partial}{\partial x_{G^\pm}} \mathcal{B}_1(x_{\nu Ji}, x_{G^\pm}) \right]. \end{aligned} \quad (48)$$

Because of the right handed neutrinos, the mass matrix of neutrino are expended to 6×6 . While, the squared mass matrix of scalar neutrinos turns to 6×6 too. The right handed neutrino contributions are very small ($10^{-15} \sim 10^{-13}$) and can be neglected safely.

B. the two-loop Barr-Zee type diagram with a closed scalar loop

The two-loop Barr-Zee type diagrams can give important contributions to muon MDM. For the exotic scalar neutrino loops, their effects are suppressed by the Higgs-lepton-lepton coupling ($\frac{m_\mu}{m_W} \sim \frac{1}{1000}$), but are enhanced by the exotic scalar neutrino-higgs-exotic scalar lepton coupling including $A_{N_4}, A_{N_5}, A_{E_4}, A_{E_5}$. The concrete expressions can be found in Eqs.(30-35). One can find the detailed discussion of ultraviolet properties for these type diagrams' contributions to muon MDM in Ref.[12].

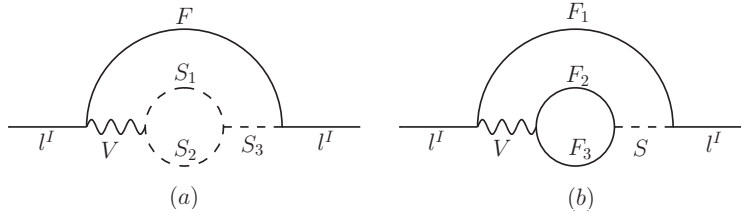


FIG. 2: The two loop Barr-Zee type diagrams with sub Fermion loop and sub scalar loop.

1. The virtual particles in the scalar loop are all neutral particles

The closed scalar virtual particles are neutral Higgs, scalar neutrinos and exotic scalar neutrinos, and they are attached to Z (neutral Higgs). With CP-even Higgs between the scalar loop and Fermion line, the two loop diagrams Fig.2(a) with scalar neutrinos (exotic scalar neutrinos) give contributions to lepton MDM as

$$a_2^{\tilde{\nu}, H^0}(Z) = -\frac{e^3(1-4s_W^2)x_{l^I}}{4s_W^3c_W^2\sqrt{x_W}} \sum_{S=\tilde{\nu}, \tilde{N}'_{4,5}} \sum_{i,j,k=1}^2 \text{Re}[(Z_S^\dagger)^{i1} Z_S^{1j} \frac{H_{H_k^0 S_i S_j}}{M_{NP}} Z_R^{1k}] \times (4\mathcal{P}_1 + x_{l^I} \mathcal{P}_2)(x_Z, x_{H_k^0}, x_{l^I}, x_{S_i}, x_{S_j}), \quad (49)$$

The couplings $H_{H_k^0 S_i S_j}$ between CP-even Higgs and exotic scalar neutrinos($\tilde{N}'_{4,5}$), can be found in Eqs.(32), (33). Because the super fields \tilde{N}^c is introduced in BLMSSM, the couplings related with the MSSM scalar neutrinos are changed, and they are corrected as

$$\begin{aligned} H_{H_1^0 \tilde{\nu}_i \tilde{\nu}_j} &= (N_M^u)_{ij} \sin \alpha + (N_M^d)_{ij} \cos \alpha; & H_{H_2^0 \tilde{\nu}_i \tilde{\nu}_j} &= (N_M^u)_{ij} \cos \alpha - (N_M^d)_{ij} \sin \alpha, \\ (N_M^u)_{ij} &= V_{EW} \sin \beta \left(\frac{e^2}{4s_W^2 c_W^2} (Z_\nu^\dagger)^{i1} Z_\nu^{1j} - |Y_\nu|^2 \delta_{ij} \right) + \lambda_{\nu_c}^* \bar{\nu}_L (Z_\nu^\dagger)^{i2} Z_\nu^{1j} - \frac{A_N}{\sqrt{2}} (Z_\nu^\dagger)^{i2} Z_\nu^{1j}, \\ (N_M^d)_{ij} &= -\frac{e^2}{4s_W^2 c_W^2} V_{EW} \cos \beta (Z_\nu^\dagger)^{i1} Z_\nu^{1j} - \frac{\mu^*}{\sqrt{2}} Y_\nu. \end{aligned} \quad (50)$$

The concrete forms of the functions $\mathcal{P}_1, \mathcal{P}_2$ are

$$\begin{aligned}\mathcal{P}_1(v, t, f, s, w) &= \frac{f(s-w)}{8} \frac{\partial^2}{\partial f^2} \left(\mathcal{B}(s, w) \mathcal{C}(v, t, f) + \mathcal{F}(v, t, f, s, w) + \frac{\mathcal{C}_1(v, t, f)}{16\pi^2} \right), \\ \mathcal{P}_2(v, t, f, s, w) &= -\frac{4}{3} \left(2 + f \frac{\partial}{\partial f} \right) \frac{1}{f} \mathcal{P}_1(v, t, f, s, w).\end{aligned}\quad (51)$$

The functions $\mathcal{C}, \mathcal{C}_1, \mathcal{F}$ are collected in the appendix.

In this type, when the scalar particles between the scalar loop and Fermion line are CP-odd Higgs, the contributions from scalar neutrinos (exotic scalar neutrinos) read as

$$\begin{aligned}a_2^{\tilde{\nu}, A^0}(Z) &= -\frac{e^3 x_{l^I}}{4s_W^3 c_W^2 \sqrt{x_W}} \sum_{S=\tilde{\nu}, \tilde{N}'} \sum_{i,j,k=1}^2 \text{Re}[Z_H^{1k} (Z_S^\dagger)^{i1} Z_S^{1j} \frac{H_{A_k^0 S_i S_j}}{M_{NP}}] \\ &\times (4\mathcal{P}_1 - x_{l^I} \mathcal{P}_2)(x_Z, x_{A_k^0}, x_{l^I}, x_{S_i}, x_{S_j}).\end{aligned}\quad (52)$$

One can find CP-odd Higgs and exotic scalar neutrinos (\tilde{N}') couplings $H_{A_k^0 S_i S_j}$ in Eq.(35). The concrete forms for the couplings between CP-odd Higgs and MSSM scalar neutrinos are corrected as

$$\begin{aligned}H_{A_1^0 \tilde{\nu}_i \tilde{\nu}_j} &= \cos \beta (P_N^u)_{ij} - \sin \beta (P_N^d)_{ij}, \quad (P_N^d)_{ij} = -\frac{\mu^*}{\sqrt{2}} Y_\nu (Z_\nu^\dagger)^{i2} Z_\nu^{1j}, \\ H_{A_2^0 \tilde{\nu}_i \tilde{\nu}_j} &= \sin \beta (P_N^u)_{ij} + \cos \beta (P_N^d)_{ij}, \quad (P_N^u)_{ij} = -(\lambda_{\nu_c}^* \bar{\nu}_L - \frac{A_N}{\sqrt{2}}) (Z_\nu^\dagger)^{i2} Z_\nu^{1j}.\end{aligned}\quad (53)$$

When the scalar particles in the scalar loop are all neutral Higgs, the corrections to lepton MDM from Fig.2(a) are

$$\begin{aligned}a_2^{H^0, A^0}(Z) &= \frac{-e^5 x_{l^I}}{16s_W^5 c_W^4 \sqrt{x_W}} \sum_{i,j,k=1}^2 \text{Re}[\frac{B_R^i}{M_{NP}} A_M^{ij} A_H^{jk} Z_H^{1k}] \\ &\times (4\mathcal{P}_1 - x_{l^I} \mathcal{P}_2)(x_Z, x_{A_k^0}, x_{l^I}, x_{H_i^0}, x_{A_j^0}),\end{aligned}\quad (54)$$

where the couplings $B_R^i, A_M^{ij}, A_H^{jk}$ can be found in Ref.[5].

2. The vector is γ (Z), and the scalar loop are charged scalar particles

When the vector is photon, contributions from the Fig.2(a) are just produced by the neutral CP even Higgs. That is to say the corresponding CP odd Higgs' contribution is zero.

$$\begin{aligned}a_2^{S, H^0}(\gamma) &= -\frac{8Q_S e^3 x_{l^I}}{s_W \sqrt{x_W}} \sum_{i=1}^2 \left(\sum_{S=\tilde{L}, \tilde{U}, \tilde{D}, \tilde{L}', \tilde{U}', \tilde{D}', H^\pm} \right) \text{Re}[Z_R^{1i} \frac{H_{H_i^0 S S}}{M_{NP}}] \\ &\times \left((Q_S \mathcal{W}_1 + \mathcal{P}_1) + x_{l^I} (Q_S \mathcal{W}_2 + \frac{1}{4} \mathcal{P}_2) \right) (0, x_{l^I}, x_{H_i^0}, x_S, x_S).\end{aligned}\quad (55)$$

Here, Q_S is the electric charge of the scalar particles. One can find the functions $\mathcal{W}_1, \mathcal{W}_2$ in the appendix. $H_{H_i^0 SS}$ are the couplings for CP even Higgs and two scalar particles. With S representing the MSSM particles $\tilde{L}, \tilde{U}, \tilde{D}, H^\pm$, the concrete forms of $H_{H_i^0 SS}$ are in Ref.[5]. We have deduced the coupling between Higgs and exotic scalar quarks in our previous work[8]. The couplings for charged exotic scalar leptons and neutral Higgs are shown in Eq.(32). The exotic scalar quark loop contributions are also suppressed by the factor $\frac{m_\mu}{m_W} \sim \frac{1}{1000}$, and they are increased by the coupling of Higgs-exotic scalar quarks-exotic scalar quarks which includes $A_{d4}, A_{d5}, A_{u4}, A_{u5}$ et al. Their concrete forms can be found in Eqs.(36-38).

For the Fig.2(a), with vector Z , both CP even and CP odd Higgs give corrections to the lepton MDM, and their results are obtained here.

$$\begin{aligned}
a_2^{S, H^0}(Z) &= \frac{e^3 x_{lI}}{s_W^3 c_W^2 \sqrt{x_W}} \sum_{i=1}^2 \left(\sum_{S_1, S_2 = \tilde{L}, \tilde{U}, \tilde{D}, \tilde{L}', \tilde{U}', \tilde{D}', H^\pm} \right) \left[\text{Re} \left[\frac{H_{H^0 S_1 S_2}}{M_{NP}} G_{Z S_1 S_2} Z_R^{1i} \right] \right. \\
&\times (1 - 4s_W^2) \left((Q_S \mathcal{W}_1 + \mathcal{P}_1) + x_{lI} (Q_S \mathcal{W}_2 + \frac{\mathcal{P}_2}{4}) \right) (x_Z, x_{lI}, x_{H_i^0}, x_{S_1}, x_{S_2}) \\
&\left. + \text{Re} \left[\frac{H_{A^0 S_1 S_2}}{M_{NP}} G_{Z S_1 S_2} Z_H^{1i} \right] \left((Q_S \mathcal{W}_1 + \mathcal{P}_1) - x_{lI} (Q_S \mathcal{W}_2 + \frac{\mathcal{P}_2}{4}) \right) (x_Z, x_{lI}, x_{A_i^0}, x_{S_1}, x_{S_2}) \right], \quad (56)
\end{aligned}$$

$H_{A^0 S_1 S_2} (G_{Z S_1 S_2})$ are the couplings for two scalar particles and CP odd Higgs (Z). When S_1, S_2 are MSSM scalar particles, $H_{A^0 S_1 S_2} (G_{Z S_1 S_2})$ can be found in Ref.[5]. The scalar exotic charged lepton and CP-odd Higgs (Z) coupling are in Eqs.(35), (27). Eq.(38) gives out Z and two exotic scalar quarks coupling.

3. The vector is W^\pm , and the scalar loop has charged particles

Charged scalar lepton and scalar quark in MSSM contribute to lepton MDM. In the same way, we also obtain the exotic scalar lepton and exotic scalar quark contributions.

$$\begin{aligned}
a_2^{S, H^\pm}(W^\pm) &= -\frac{4e^2}{s_W^2} \sum_{\alpha, i=1}^2 \sum_{S_1 = \tilde{L}, \tilde{L}', \tilde{D}, \tilde{D}', H^\pm} \sum_{S_2 = \tilde{\nu}, \tilde{N}', \tilde{U}, \tilde{U}', H^0, A^0} \left(\sqrt{x_{lI}} \text{Re} [G_{W S_1 S_2} (U_{\nu J}^\dagger)^{\alpha 1}] \right. \\
&\times \frac{H_{H_i^\pm S_1 S_2}}{M_{NP}} A_{H_i^\pm \nu l} \mathcal{W}_3 \left. \right) (x_\nu, x_W, x_{H_i^\pm}, x_{S_1}, x_{S_2}, Q_{S_1}, Q_{S_2}). \quad (57)
\end{aligned}$$

The complex functions $\mathcal{W}_3, \mathcal{W}_4$ are collected in the appendix. The concrete forms for the couplings related with neutrino and scalar neutrino are different from those in MSSM.

$$A_{H^\pm \nu l} = \sin \beta Y_l^I U_{\nu l J}^{1i}, \quad B_{H^\pm \nu l} = \cos \beta Y_\nu^* W_{\nu l J}^{2i}, \quad A_{G^\pm \nu l} = \cos \beta Y_l^I U_{\nu l J}^{1i},$$

$$\begin{aligned}
B_{G^\pm \nu l} &= \sin \beta Y_\nu^* W_{\nu l}^{2i}, \quad H_{H^\pm \tilde{L} \tilde{\nu}} = (L_M^u)_{ij} \cos \beta - (L_M^d)_{ij} \sin \beta, \\
H_{G^\pm \tilde{L} \tilde{\nu}} &= (L_M^u)_{ij} \sin \beta + (L_M^d)_{ij} \cos \beta, \quad G_{W \tilde{L} \tilde{\nu}} = (Z_{\nu l}^\dagger)^{i1} Z_L^{IJ},
\end{aligned} \tag{58}$$

with

$$\begin{aligned}
(L_M^u)_{ij} &= \left(-\frac{e^2}{2\sqrt{2}s_W^2} + |Y_\nu|^2\right) V_{EW} \sin \beta (Z_\nu^\dagger)^{i1} Z_{\tilde{L}}^{1j} - Y_e^* \mu (Z_\nu^\dagger)^{i1} Z_{\tilde{L}}^{2j} \\
&+ \frac{V_{EW} \cos \beta}{\sqrt{2}} Y_e^* Y_\nu (Z_\nu^\dagger)^{i2} Z_{\tilde{L}}^{2j} + (A_N - \sqrt{2} \lambda_{\nu c}^* \bar{v}_L) (Z_\nu^\dagger)^{i2} Z_{\tilde{L}}^{1j}, \\
(L_M^d)_{ij} &= \left(-\frac{e^2}{2\sqrt{2}s_W^2} + |Y_e|^2\right) V_{EW} \cos \beta (Z_\nu^\dagger)^{i1} Z_{\tilde{L}}^{1j} - Y_\nu \mu^* (Z_\nu^\dagger)^{i2} Z_{\tilde{L}}^{1j} \\
&+ \frac{V_{EW} \sin \beta}{\sqrt{2}} Y_e^* Y_\nu (Z_\nu^\dagger)^{i2} Z_{\tilde{L}}^{2j} + A_E^* (Z_\nu^\dagger)^{i1} Z_{\tilde{L}}^{2j},
\end{aligned} \tag{59}$$

The other necessary concrete forms for the couplings $G_{WS_1 S_2}$, $H_{H_i^\pm S_1 S_2}$ can be found in Eqs.(27),(38),(30),(31),(36),(37) and Ref.[8].

C. the two-loop Barr-Zee type diagram with a closed Fermion loop

When the inserted is a fermion loop, the diagrams can be divided into two parts, according to the vector neutral Boson(γ, Z) and charged one(W^\pm). For $H^\pm F_1 F_2$ coupling, it becomes large with the heavy Fermion mass, and may give important contributions.

1. the vector is γ, Z , and the Fermion loop are all charged particles

When the fermion loop is quarks and exotic quarks, charged leptons and exotic charged leptons, the contributions for muon MDM from the two loop diagrams with charged fermion loop inserted between γ and CP even Higgs are

$$\begin{aligned}
a_2^{F, H^0}(\gamma) &= \frac{16e^3 x_{lI}}{s_W \sqrt{x_W}} \sum_{i=1}^2 \sum_{F=b, t, \chi^\pm, \tau, L', b', t'} Q_F^2 Z_R^{1i} \text{Re}[Y_{H_i^0 FF}] \\
&\times \sqrt{x_F} (\mathcal{W}_5 + \mathcal{W}_6 + 2x_{lI} \mathcal{W}_7)(0, x_{lI}, x_{H_i^0}, x_F, x_F),
\end{aligned} \tag{60}$$

where $Y_{H_i^0 FF}$ are the right hand parts of the couplings between the CP-even Higgs and the Fermions($b, t, \chi^\pm, \tau, L', b', t'$), and the general form is written as $i(J_{H_i^0 FF} \omega_- + Y_{H_i^0 FF} \omega_+)$. The concrete forms of $Y_{H_i^0 FF}$ with $F = (b, t, \chi^\pm, \tau, L', b', t')$ can be found in Ref.[5, 8]. $Y_{H_i^0 FF}$ for $F = L'$ are shown in Eq.(23). To save space in the text, the form factors $\mathcal{W}_{5,6,\dots,19}$ are shown in the appendix.

In the same way, we get the two loop contributions with γ , CP odd Higgs and charged fermion loop.

$$a_2^{F,A^0}(\gamma) = \frac{16e^3 x_{l^I}}{s_W \sqrt{x_W}} \sum_{i=1}^2 \sum_{F=b,t,\chi^\pm,\tau,L',b',t'} Q_F^2 \text{Re}[Y_{A_i^0 FF} Z_H^{1i}] \times \sqrt{x_F} (\mathcal{W}_8 + \mathcal{W}_9 - x_{l^I} \mathcal{W}_{10})(0, x_{l^I}, x_{A_i^0}, x_F, x_F). \quad (61)$$

$Y_{A_i^0 FF}$ are the right hand parts of the couplings between the CP-odd Higgs with $F = (b, t, \chi^\pm, \tau, b', t')$, whose forms are obtained in the same way as $Y_{H_i^0 FF}$.

When the vector is Z instead of γ , the corresponding expressions of the MDM contribution are more complex. The results from the CP-even Higgs, Z and charged fermion loop at two loop level are

$$\begin{aligned} a_2^{F,H^0}(Z) = & 2 \frac{-Q_F e^3 x_{l^I}}{s_W^2 c_W \sqrt{x_W}} \sum_{i=1}^2 \sum_{F=b,t,\chi^\pm,\tau,L',b',t'} Z_R^{1i} \left\{ \text{Re}[(H_{ZF_1 F_2} Y_{H_i^0 F_1 F_2} \right. \\ & - J_{H_i^0 F_1 F_2} T_{ZF_1 F_2})] \sqrt{x_{F_1}} \left(\mathcal{W}_8 + x_{l^I} \frac{1}{2} \mathcal{W}_{10} \right) (x_Z, x_{l^I}, x_{H_i^0}, x_{F_1}, x_{F_2}) \\ & + \text{Re}[(T_{ZF_1 F_2} Y_{H_i^0 F_1 F_2} - H_{ZF_1 F_2} J_{H_i^0 F_1 F_2})] \sqrt{x_{F_2}} \left(\mathcal{W}_9(x_Z, x_{l^I}, x_{H_i^0}, x_{F_1}, x_{F_2}) \right. \\ & + x_{l^I} \frac{1}{2} \mathcal{W}_{10}(x_Z, x_{l^I}, x_{H_i^0}, x_{F_2}, x_{F_1}) \Big) + \text{Re}[(1 - 4s_W^2)(H_{ZF_1 F_2} J_{H_i^0 F_1 F_2} \\ & + T_{ZF_1 F_2} Y_{H_i^0 F_1 F_2})] \sqrt{x_{F_2}} \left(\mathcal{W}_5 + x_{l^I} \mathcal{W}_7 \right) (x_Z, x_{l^I}, x_{H_i^0}, x_{F_1}, x_{F_2}) \\ & + \text{Re}[(1 - 4s_W^2)(J_{H_i^0 F_1 F_2} T_{ZF_1 F_2} + H_{ZF_1 F_2} Y_{H_i^0 F_1 F_2})] \sqrt{x_{F_1}} \\ & \left. \times (\mathcal{W}_6(x_Z, x_{l^I}, x_{H_i^0}, x_{F_1}, x_{F_2}) + x_{l^I} \mathcal{W}_7(x_Z, x_{l^I}, x_{H_i^0}, x_{F_2}, x_{F_1})) \right\}. \quad (62) \end{aligned}$$

Generally, $ZF_1 F_2$ couplings are expressed as $ie(H_{ZF_1 F_2} \gamma_\alpha \omega_- + T_{ZF_1 F_2} \gamma_\alpha \omega_+)$. One can obtain $H_{ZF_1 F_2}, T_{ZF_1 F_2}$ for $F = (L', b', t')$ in Eqs.(26,40). $Y_{H_i^0 F_1 F_2}$ are similar with $Y_{H_i^0 FF}$, while $J_{H_i^0 F_1 F_2}$ are couplings of the left parts.

We also obtain the CP-odd Higgs contribution from the diagram with vector Z and charged fermion loop shown in Fig.2(b).

$$\begin{aligned} a_2^{F,A^0}(Z) = & 2 \frac{-Q_F e^3 x_{l^I}}{s_W^2 c_W \sqrt{x_W}} \sum_{i=1}^2 \sum_{F=b,t,\chi^\pm,\tau,L',b',t'} Z_H^{1i} \left\{ (1 - 4s_W^2) \text{Re}[(T_{ZF_1 F_2} Y_{A_i^0 F_1 F_2} \right. \\ & - H_{ZF_1 F_2} J_{A_i^0 F_1 F_2})] \sqrt{x_{F_2}} \left(\mathcal{W}_9(x_Z, x_{l^I}, x_{A_i^0}, x_{F_1}, x_{F_2}) - \frac{x_{l^I}}{2} \mathcal{W}_{10}(x_Z, x_{l^I}, x_{A_i^0}, x_{F_2}, x_{F_1}) \right) \\ & + \text{Re}[(H_{ZF_1 F_2} J_{A_i^0 F_1 F_2} + T_{ZF_1 F_2} Y_{A_i^0 F_1 F_2})] \sqrt{x_{F_2}} \left(\mathcal{W}_5 - x_{l^I} \mathcal{W}_7 \right) (x_Z, x_{l^I}, x_{A_i^0}, x_{F_1}, x_{F_2}) \\ & + \text{Re}[(J_{A_i^0 F_1 F_2} T_{ZF_1 F_2} + H_{ZF_1 F_2} Y_{A_i^0 F_1 F_2})] \sqrt{x_{F_1}} \left(\mathcal{W}_6(x_Z, x_{l^I}, x_{A_i^0}, x_{F_1}, x_{F_2}) \right. \\ & - x_{l^I} \mathcal{W}_7(x_Z, x_{l^I}, x_{A_i^0}, x_{F_2}, x_{F_1}) \Big) + (1 - 4s_W^2) \text{Re}[(H_{ZF_1 F_2} Y_{A_i^0 F_1 F_2} \\ & - J_{A_i^0 F_1 F_2} T_{ZF_1 F_2})] \sqrt{x_{F_1}} \left(\mathcal{W}_8 - x_{l^I} \frac{1}{2} \mathcal{W}_{10} \right) (x_Z, x_{l^I}, x_{A_i^0}, x_{F_1}, x_{F_2}) \Big\}. \quad (63) \end{aligned}$$

Similarly, we get the CP odd Higgs couplings $Y_{A_i^0 F_1 F_2}, J_{A_i^0 F_1 F_2}$.

D. the vector is W^\pm , and the fermion loop have charged particles

The contribution to lepton MDM from the diagram with vector W^\pm and Fermion loop are obtained here.

$$\begin{aligned}
d_2^{F, H^\pm}(W^\pm) = & \sum_{i=1}^2 \sum_{F_1=b, b', \tau, L', \chi^\pm} \sum_{F_2=t, t', \nu_\tau, N', \chi^0} \frac{4e^2}{s_W^2} \left(\text{Re}[A_{H_i^\pm \nu l}(U_{\nu J}^\dagger)^{\alpha 1} (T_{W F_1 F_2} Y_{H_i^\pm F_1 F_2} \mathcal{W}_{12} \right. \\
& + H_{W F_1 F_2} J_{H_i^\pm F_1 F_2} \mathcal{W}_{13})] \sqrt{x_{lI} x_{F_2}} + \sqrt{x_{lI} x_{F_1}} \text{Re}[A_{H_i^\pm \nu l}(U_{\nu J}^\dagger)^{\alpha 1} (H_{W F_1 F_2} Y_{H_i^\pm F_1 F_2} \mathcal{W}_{14} \\
& \left. + J_{H_i^\pm F_1 F_2} T_{W F_1 F_2} \mathcal{W}_{15})] \right) (x_W, x_\nu, x_{H_i^\pm}, x_{F_1}, x_{F_2}, Q_{F_1}, Q_{F_2}). \quad (64)
\end{aligned}$$

The couplings related with exotic leptons (quarks) are given in Eq.(22), (26), (40) and Ref.[9].

Because the right handed neutrino is introduced in BLMSSM, the couplings related with neutrino are not same as those in MSSM. We deduced the needed couplings here.

$$\begin{aligned}
H_{W\tau\nu_\tau} &= (U_{\nu IJ}^\dagger)^{i1}, \quad F_{G^\pm L\nu} = -Y_\nu \sin \beta (W_{\nu IJ}^\dagger)^{i2}, \quad Y_{G^\pm L\nu} = Y_l^{I*} \cos \beta (U_{\nu IJ}^\dagger)^{i1}, \\
T_{W\tau\nu_\tau} &= 0, \quad F_{H^\pm L\nu} = -Y_\nu \cos \beta (W_{\nu IJ}^\dagger)^{i2}, \quad Y_{H^\pm L\nu} = -Y_l^{I*} \sin \beta (U_{\nu IJ}^\dagger)^{i1}. \quad (65)
\end{aligned}$$

IV. THE NUMERICAL RESULTS

In this section, we show our numerical results. For the input parameters, we take into account the experimental constraints from the lightest neutral CP even Higgs $m_{h^0} \simeq 125.7$ GeV and the neutrino experiment data:

$$\begin{aligned}
\sin^2 2\theta_{13} &= 0.090 \pm 0.009, \quad \sin^2 \theta_{12} = 0.306_{-0.015}^{+0.018}, \quad \sin^2 \theta_{23} = 0.42_{-0.03}^{+0.08}, \\
\Delta m_\odot^2 &= 7.58_{-0.26}^{+0.22} \times 10^{-5} \text{eV}^2, \quad |\Delta m_A^2| = 2.35_{-0.09}^{+0.12} \times 10^{-3} \text{eV}^2. \quad (66)
\end{aligned}$$

In our previous work, we fit the neutrino experiment data shown as Eq.(66) in BLMSSM [20]. The lepton flavor violation is taken into account through $(Y_\nu)_{ij}, (m_{\bar{\nu}^c}^2)_{ij}, (m_L^2)_{ij}, (m_R^2)_{ij}, (i, j = 1, 2, 3)$. In the numerical discussion, the non-diagonal elements of these matrixes are not zero. Therefore, the lepton flavor violation are considered and there is a transition between muon-sneutrinos and tau-sneutrinos.

Firstly, we give out the SM relevant parameters.

$$\begin{aligned}
\alpha_s(m_Z) &= 0.118, \quad \alpha(m_Z) = 1/128, \quad s_W^2(m_Z) = 0.23, \quad m_W = 80.4\text{GeV}, \\
m_Z &= 91.2\text{GeV}, \quad m_t = 174.2\text{GeV}, \quad m_b = 4.2\text{GeV}, \quad m_u = 2.3 \times 10^{-3}\text{GeV}, \\
m_d &= 4.8 \times 10^{-3}\text{GeV}, \quad m_s = 0.95 \times 10^{-3}\text{GeV}, \quad m_c = 1.275\text{GeV}, \\
m_e &= 0.51 \times 10^{-3}\text{GeV}, \quad m_\mu = 0.105\text{GeV}, \quad m_\tau = 1.777\text{GeV}, \\
\text{CKM}_{11} &= 0.9743, \quad \text{CKM}_{22} = 0.9734, \quad \text{CKM}_{33} = 0.9991.
\end{aligned} \tag{67}$$

Because the muon MDM is related with the real parts of the results, to simplify the numerical discussion we suppose all the involved parameters in BLMSSM are real.

The used parameters in BLMSSM are collected here.

$$\begin{aligned}
\tan \beta_B &= \tan \beta_L = 2, \quad B_4 = L_4 = \frac{3}{2}, \quad \tan \beta = 15, \\
m_{\tilde{Q}_3} &= m_{\tilde{U}_3} = m_{\tilde{D}_3} = 1.4\text{TeV}, \quad m_{Z_B} = m_{Z_L} = 1\text{TeV}, \\
m_{\tilde{U}_4} &= m_{\tilde{D}_4} = m_{\tilde{Q}_5} = m_{\tilde{U}_5} = m_{\tilde{D}_5} = 1\text{TeV}, \quad m_{\tilde{Q}_4} = 790\text{GeV}, \\
m_{\tilde{L}_4} &= m_{\tilde{\nu}_4} = m_{\tilde{E}_4} = m_{\tilde{L}_5} = m_{\tilde{\nu}_5} = m_{\tilde{E}_5} = 1.4\text{TeV}, \\
A_{u_4} &= A_{u_5} = A_{d_4} = A_{d_5} = 550\text{GeV}, \quad A_b = A_t = -1\text{TeV}, \\
v_{B_t} &= \sqrt{v_B^2 + \bar{v}_B^2} = 3\text{TeV}, \quad v_{L_t} = \sqrt{v_L^2 + \bar{v}_L^2} = 3\text{TeV}, \\
m_1 &= 1\text{TeV}, \quad m_2 = 750\text{GeV}, \quad \mu_H = -800\text{GeV}. \\
Y_{u_4} &= 0.8Y_t, \quad Y_{d_4} = 0.7Y_b, \quad Y_{u_5} = 0.7Y_b, \quad Y_{d_5} = 0.1Y_t, \\
A'_e &= A'_\mu = A'_\tau = 130\text{GeV}, \quad \lambda_{\nu^c} = 1, \quad A_{\nu_4} = A_{\nu_5} = 550\text{GeV}, \\
A'_u &= A'_c = A'_t = A'_d = A'_s = A'_b = 500\text{GeV}, \\
Y_{\nu_4} &= 0.6, \quad Y_{\nu_5} = 1.1, \quad Y_{e_4} = 1.3, \quad Y_{e_5} = 0.6, \quad \mu_L = 500\text{GeV} \\
A_{\nu_e} &= A_{\nu_\mu} = A_{\nu_\tau} = A_{\nu_e^c} = A_{\nu_\mu^c} = A_{\nu_\tau^c} = -500\text{GeV}.
\end{aligned} \tag{68}$$

We suppose the following relations in the numerical discussion, then the numerical discussion is simplified.

$$\begin{aligned}
A_e &= A_\mu = A_\tau = A_L, \quad A_c = A_s = A_{cs}, \\
m_{\tilde{e}} &= m_{\tilde{\mu}} = m_{\tilde{\tau}} = m_{\tilde{\nu}_e} = m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_\tau} = ML_s \\
A_{e_4} &= A_{e_5} = AE_{45}, \quad m_{\tilde{Q}_2} = m_{\tilde{U}_2} = m_{\tilde{D}_2} = MQ_2;
\end{aligned} \tag{69}$$

In order to reflect the flavor mixing obviously and simplify the discussion, we define the off-diagonal elements in the following form.

$$(m_{\tilde{\nu}^c}^2)_{ij} = (m_L^2)_{ij} = (m_R^2)_{ij} = MLa^2, \quad (AL)_{ij} = ALa, \quad \text{with } i \neq j, (i, j = 1, 2, 3). \quad (70)$$

When $MLa = 0$, $ALa = 0$ there is no flavor mixing for the scalar leptons.

A. the one loop numerical results

Comparing with the two loop contributions from the new physics, the one loop new physics contributions to muon MDM are dominant. Therefore, the parameters having relation with the one loop contributions should affect the results apparently. ML_s is in the squared mass matrixes of scalar charged leptons and scalar neutrinos, and these scalar particles can give one loop corrections to muon MDM.

At first, supposing $MLa = 0$ and $ALa = 0$, we study the one loop contributions individually with the varying ML_s (500-2000 GeV). The one loop scalar lepton-neutralino contributions are plotted in Fig.3.

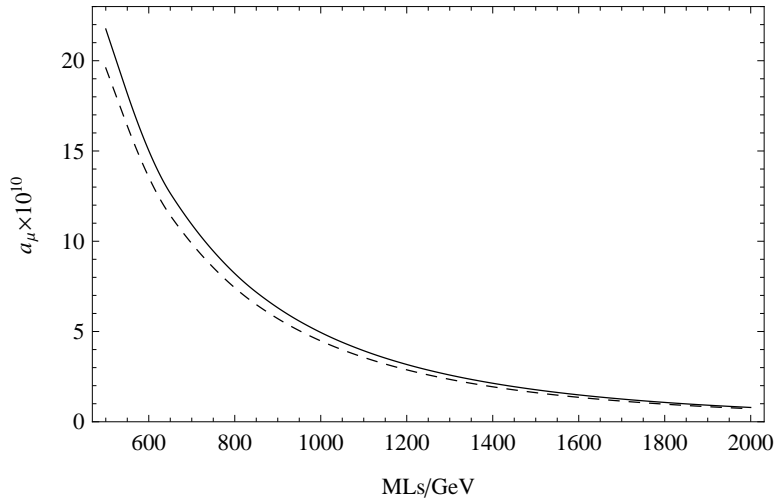


FIG. 3: The one loop scalar lepton and neutralino contributions to muon MDM, the dashed-line and solid-line represent muon MDM varying with ML_s , for $A_L = -500\text{GeV}$ and $A_L = -800\text{GeV}$ respectively.

The one loop scalar lepton-neutralino contributions are about 4.5×10^{-10} when $ML_s = 1000\text{GeV}$. The contributions turn large with the decreasing ML_s . They can reach 20×10^{-10}

with $MLs = 500\text{GeV}$. If MLs turns smaller than 500GeV , these contributions can be much larger. As $MLs > 1600\text{ GeV}$, the results turn small, which are about 1.7×10^{-10} . The value of A_L affects the results slightly. From Fig.3, one can find the dotted line is up the solid line to small extent.

The one loop scalar neutrino-chargino contributions are plotted in Fig.4. The dashed

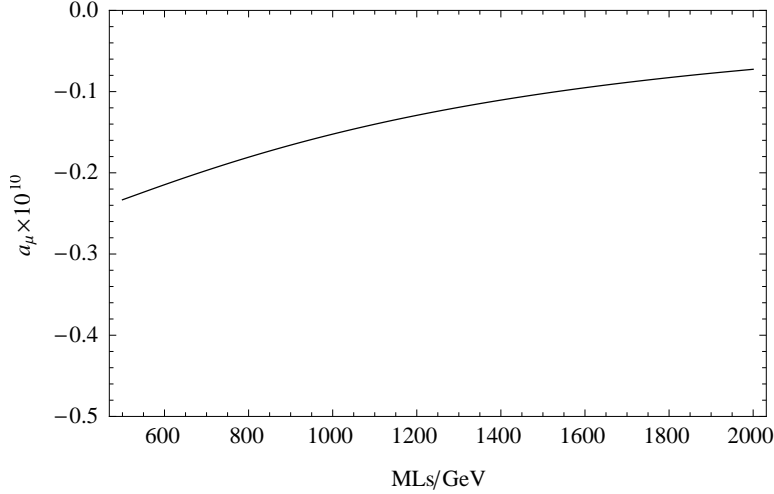


FIG. 4: The one loop scalar neutrino-chargino contributions to muon MDM, the solid-line and dashed-line represent muon MDM varying with MLs , for $A_L = -500\text{GeV}$ and $A_L = -800\text{GeV}$ respectively.

line and solid line are coincident. The scalar neutrino-chargino one loop contributions are about -2.0×10^{-11} which are approximately one order smaller than the one loop scalar lepton-neutralino contributions. Obviously, these results vary slightly with MLs .

The neutral Higgs-lepton and charged Higgs-neutrino one loop contributions are both at the order of 10^{-14} . Therefore, one should not consider them. Then the one loop scalar lepton-neutralino contributions are dominant. In MSSM, the one loop contribution to muon MDM is approximately

$$13 \times 10^{-10} \left(\frac{100\text{GeV}}{M_{SUSY}} \right)^2 \tan \beta \text{sign}(\mu_H). \quad (71)$$

With $M_{SUSY} = 1000\text{ GeV}$, and $\tan \beta = 15$, the MSSM one loop contribution to muon MDM is about 2.0×10^{-10} . In our results, when $\tan \beta = 15$ and $MLs = 1000\text{ GeV}$, the BLMSSM one loop result is about 4.5×10^{-10} . Roughly speaking, the BLMSSM one loop result accords with MSSM one loop estimate. Strictly speaking, the BLMSSM one loop result is

about double as MSSM one loop estimate. What is the reason? When $MLs = 1000$ GeV, half of the scalar lepton masses are about 800GeV, the others are about 1000GeV, and the neutralinos masses are about 700GeV, which enhance the BLMSSM contributions. So using MSSM estimate formula with $M_{SUSY} \sim 700$ GeV we can obtain the one loop contribution 4.0×10^{-10} . On the whole, the BLMSSM one loop results confirm with the one loop MSSM estimate.

In order to embody the flavor mixing effects, we suppose $MLa \neq 0$. With $ALa = 0$ and $MLs = 600(800, 1000)$ GeV, we plot the results with the varying MLa . Fig.5 represents the relation between one loop scalar lepton-neutralino contribution and MLs, MLa .

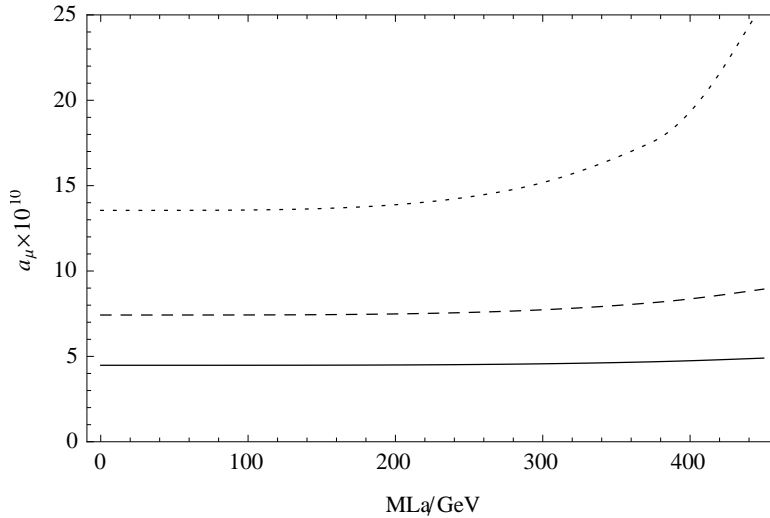


FIG. 5: The one loop scalar lepton-neutralino contributions to muon MDM, the dotted-line, dashed-line and solid-line represent muon MDM varying with MLa , for $MLs = 600$ GeV, $MLs = 800$ GeV and $MLs = 1000$ GeV respectively.

The solid line is the result with $MLs = 1000$ GeV, and MLa varies from 0 to 450GeV. MLa affects the solid line weakly, but one can still find the result is increasing function of MLa . With $MLs = 1000$ GeV, the one loop scalar lepton-neutralino contribution to muon MDM is around 4.5×10^{-10} . The dashed line representing result with $MLa = 800$ GeV, and the level influenced by MLa is a little stronger than that of the solid line. The dashed line implies the result is about 8.0×10^{-10} . The dotted line is obtained with $MLs = 600$ GeV, and it is strongly affected by MLa . $MLa = 0$, the dotted line is about 14×10^{-10} . When $MLa > 300$ GeV, the dotted line increases quickly. With $MLa = 400$ GeV, the dotted line can reach 20×10^{-10} and even larger. These results imply the flavor mixing can enhance

the contributions. The flavor mixing enhance extent can be approximately expressed by the ratio between off-diagonal elements and diagonal elements for $(m_{\nu^c}^2)_{IJ}, (m_L^2)_{IJ}, (m_R^2)_{IJ}$, whose concrete form is $\frac{MLa^2}{MLs^2}$.

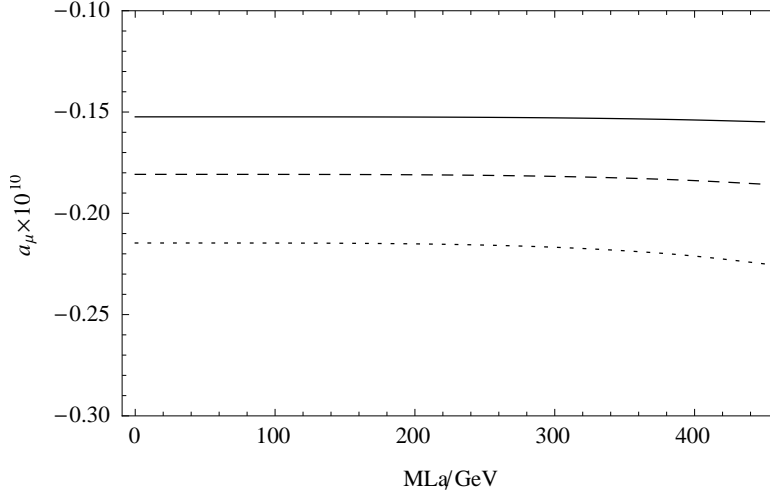


FIG. 6: The one loop scalar neutrino-chargino contributions to muon MDM, the dotted-line, dashed-line and solid-line represent muon MDM varying with ML_a , for $MLs = 600\text{GeV}$, $MLs = 800\text{GeV}$ and $MLs = 1000\text{GeV}$ respectively.

In Fig.6, the one loop scalar neutrino-chargino contributions to muon MDM are obtained. The solid line corresponds to $MLs = 1000\text{ GeV}$, and the result is about -1.5×10^{-11} . The dashed line result with $MLs = 800\text{ GeV}$ is about -1.7×10^{-10} . The dotted line representing $MLs = 600\text{ GeV}$ result, and it is around -2.2×10^{-11} . These three lines vary weakly with ML_a . However, we also can find that the extent affected by ML_a for the three lines follow the same rule: dotted-line > dashed-line > solid-line.

We also calculate the contribution from the off-diagonal element ALa . The numerical results imply that the effect of ALa is very small for both the one loop scalar lepton-neutralino contribution and the scalar neutrino-chargino contribution. Therefore, one can neglect ALa safely, and in the latter numerical study, we suppose $ALa = 0$.

B. the sum of one loop and the two loop results

Supposing, $AE_{45} = 550\text{GeV}$ and $A_{cs} = -500\text{GeV}$, $MQ_2 = 1000\text{GeV}$, in Fig.7 we plot the muon MDM varying with ML_s for $A_L = -500\text{GeV}$ and $A_L = -800\text{GeV}$ respectively.

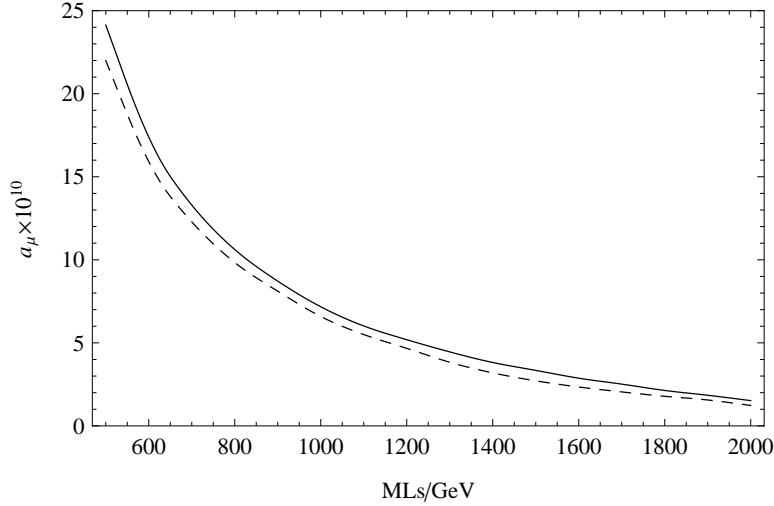


FIG. 7: As $AE_{45} = 550\text{GeV}$ and $A_{cs} = -500\text{GeV}$, $MQ_2 = 1000\text{GeV}$, the dashed-line and solid-line represent muon MDM varying with ML_s , for $A_L = -500\text{GeV}$ and $A_L = -800\text{GeV}$ respectively.

The solid line is on the dashed line. It implies when $A_L = -800\text{GeV}$, the numerical results are larger than the corresponding results with $AL = -500\text{GeV}$. When $ML_s < 1000\text{GeV}$, the numerical results turn large quickly with the decrescent ML_s . In the ML_s region $500 \sim 600\text{GeV}$, the new physics contributions reach 20×10^{-10} , and the values can be even larger. The large value is able to remedy the deviation between the SM prediction and experiment result for muon MDM well. With the enlarging ML_s , the one loop results decrease obviously. However, the two loop contributions having no relation with ML_s do not change. That is to say the importance of the two loop contributions turns large when the one loop contributions decrease with the enlarging ML_s .

In Ref.[13], authors study some two loop diagrams in MSSM, where a loop of charginos or neutralinos, the superpartners of gauge and Higgs, is inserted into a two-Higgs-doublet one-loop diagram. Their numerical results can reach 10×10^{-10} , which is large. They also study the two loop SUSY corrections to muon MDM from the diagrams with a closed scalar fermion or fermion loop and/or Higgs boson exchange. These contributions are in the region of $0.5\sigma \sim 3\sigma$. Our two loop results are at the order of 10^{-10} . With the used parameters in BLMSSM, our studied two loop results vary in the region of $0.5 \sim 4.0 \times 10^{-10}$.

When the sub-scalar loop particles are Higgs(charged Higgs) and the virtual vectors are γ, Z, W , these type two loop contributions are small $\sim 10^{-14}$ and even smaller. The scalar neutrino loop and exotic scalar neutrino loop contributions are at the order of 10^{-11} . The

contributions from the scalar leptons and scalar quarks are of $10^{-11} \sim 10^{-10}$ order. The exotic scalar quark contributions is at the order of $10^{-12} \sim 10^{-11}$, and it is smaller than the exotic scalar lepton contributions $10^{-11} \sim 10^{-10}$.

For the sub-Fermion loop, the corrections from the SM particles (τ, b, t) are small $10^{-15} \sim 10^{-12}$, because of the Fermion-Fermion-Higgs coupling. In this condition, to obtain considerable contributions the fermion should be heavy to enhance the Fermion-Fermion-Higgs coupling. However, when the virtual particles are all very heavy, their contributions will be suppressed. In the numerical results the two loop neutralino and chargino contributions are at the order of $10^{-11} \sim 10^{-10}$. While, the contributions from the exotic leptons, exotic quarks and exotic neutrinos are at the order of $10^{-12} \sim 10^{-11}$.

The parameters MQ_2 relates with the square mass matrix of the 2nd generation scalar quarks, and it's contribution to muon MDM appears at two loop level. Therefore, it's effect should smaller than that of ML_s . Taking $AE_{45} = 550\text{GeV}$, $A_{cs} = -500\text{GeV}$, $A_L = -800\text{GeV}$ and $ML_s = 500(800)\text{GeV}$, the numerical results are obtained with in Fig.8, which shows the muon MDM varying with MQ_2 very mildly. The dashed line represents the result for $ML_s = 800\text{GeV}$, and is about 10.5×10^{-10} . On the other hand, the result shown as the solid line is around 24×10^{-10} . The BLMSSM corrections decrease weakly with the enlarging MQ_2 .

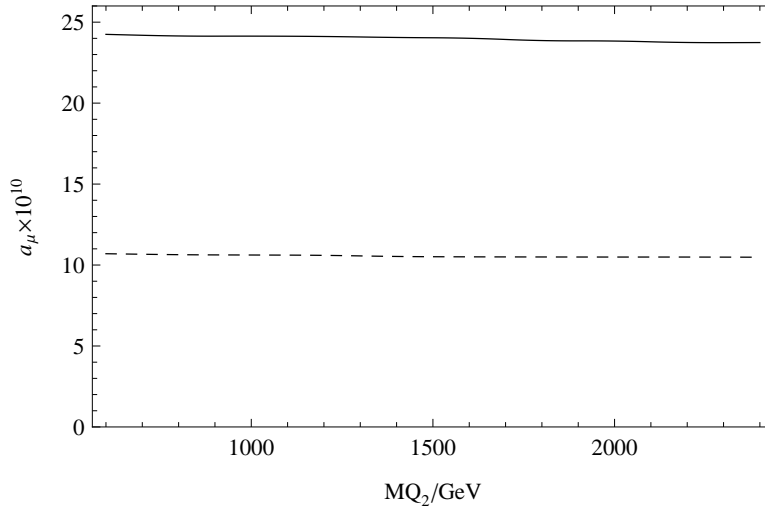


FIG. 8: As $AE_{45} = 550\text{GeV}$ and $A_{cs} = -500\text{GeV}$, $AL = -800\text{GeV}$, the solid-line and dashed-line represent muon MDM varying with MQ_2 , for $ML_s = 500\text{GeV}$ and $ML_s = 800\text{GeV}$ respectively.

The squared mass matrixes of the charged exotic scalar leptons contain the parameters

AE_{45} . With $A_{cs} = -500\text{GeV}$, $MQ_2 = 1000\text{GeV}$, $A_L = -800\text{GeV}$, we plot the results versus AE_{45} for $ML_2 = 500\text{GeV}$ and $ML_2 = 800\text{GeV}$ respectively. From Fig.9, one finds that the AE_{45} affects the results slightly. When $ML_2 = 800\text{GeV}$, the corrections are about 10.5×10^{-10} . As while as, the corrections reach 24×10^{-10} with $ML_2 = 500\text{GeV}$. The AE_{45} effect to muon MDM is in the region $10^{-12} \sim 10^{-11}$. However, we still can see that the correction is the very slowly increasing function of AE_{45} .

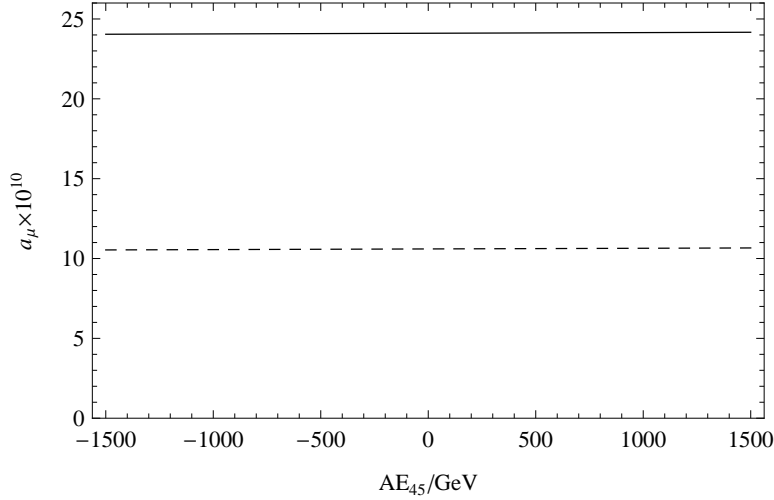


FIG. 9: As $A_L = -800\text{GeV}$, $A_{cs} = -500\text{GeV}$, $MQ_2 = 1000\text{GeV}$, the solid-line and dashed-line represent muon MDM varying with AE_{45} , for $ML_s = 500\text{GeV}$ and $ML_s = 800\text{GeV}$ respectively.

In Fig.10, we plot the results versus A_{cs} for $ML_s = 500(700, 900)\text{GeV}$ with $AE_{45} = 550\text{GeV}$, $AL = -800\text{GeV}$, $MQ_2 = 1000\text{GeV}$. The solid line is obtained with $ML_s = 500\text{GeV}$, and the result is about 24×10^{-10} . The dashed line representing the correction with $ML_s = 700\text{GeV}$ is around 13.5×10^{-10} . For $ML_s = 900\text{GeV}$, the correction is about 8.5×10^{-10} . The three lines all turn weakly with the varying A_{cs} . From Figs.(8,9,10), one can easily find that the parameters just relating with the two loop contributions to muon MDM have small influence to the results, because the one loop contribution is dominant.

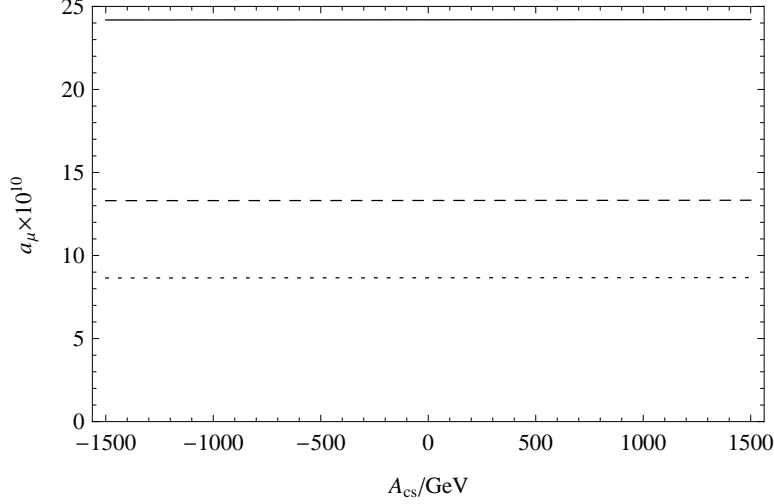


FIG. 10: As $AE_{45} = 550\text{GeV}$, $A_L = -800\text{GeV}$, $MQ_2 = 1000\text{GeV}$, the solid-line, dashed-line and dotted line represent muon MDM varying with A_{cs} , for $ML_s = 500(700, 900)\text{GeV}$ respectively.

V. DISCUSSION AND CONCLUSION

In the framework of the BLMSSM, the muon MDM is studied in this work. We calculate the one loop diagrams and the Barr-Zee type two loop diagrams. In the numerical analysis, we consider the experiment constraints such as: the experiment data of the lightest CP-even Higgs and neutrino. Our numerical results imply when the exotic and SUSY particles are not very heavy such as at TeV scale, the new physics contribution is about 8.0×10^{-10} and even larger. In the parameter space as we supposed, our numerical results can reach 24×10^{-10} , as scalar leptons at 500GeV scale, which can well remedy the deviation between the experiment data and the SM theoretical prediction for muon MDM.

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Appendix A: the functions

The one-loop, two-loop functions and the form factors are collected here.

$$\begin{aligned}
\mathcal{C}(x, y, z) &= -\frac{1}{16\pi^2} \left(\frac{x \log(x)}{(x-y)(x-z)} + \frac{y \log(y)}{(y-x)(y-z)} + \frac{z \log(z)}{(x-z)(y-z)} \right) \\
\mathcal{C}_1(x, y, z) &= \frac{1}{32\pi^2} \left(\frac{x \log^2(x)}{(x-y)(x-z)} + \frac{y \log^2(y)}{(x-y)(z-y)} + \frac{z \log^2(z)}{(z-x)(z-y)} \right) \\
\mathcal{F}(v, f, t, s, w) &= \frac{1}{512\pi^4} \left(\frac{\mathcal{G}(f, s, w)}{(f-t)(f-v)} + \frac{\mathcal{G}(t, s, w)}{(f-t)(v-t)} + \frac{\mathcal{G}(v, s, w)}{(v-f)(v-t)} \right) \\
\mathcal{G}(x, y, z) &= -\Phi(x, y, z) - 2(x+y+z) + 2(x \log(x) + y \log(y) + z \log(z)) \\
&\quad - x \log^2(x) - y \log^2(y) - z \log^2(z). \\
\mathcal{A}(x) &= -\frac{x \log(x)}{16\pi^2}, \quad \mathcal{F}_1(v, f, t, s) = \frac{\mathcal{G}(s, f, t) - \mathcal{G}(v, f, t)}{512\pi^4(s-v)}. \tag{A1} \\
\mathcal{W}_1(v, f, t, s, w) &= \frac{1}{24} \left((4w-t) \frac{\partial^2}{\partial t \partial s} + (4s-t) \frac{\partial^2}{\partial t \partial w} - \frac{\partial}{\partial s} - \frac{\partial}{\partial w} \right) \mathcal{F}_1(f, s, w, t) + \frac{1}{8} \frac{\partial}{\partial s} \mathcal{F}_1(f, s, w, v) \\
&\quad + \frac{1}{24} \left[(-t+4w-4s) \frac{\partial \mathcal{A}(s)}{\partial s} + (4s-t-4w) \frac{\partial \mathcal{A}(w)}{\partial w} - \frac{7t}{16\pi^2} \frac{\partial}{\partial t} \mathcal{C}(v, f, t) + \frac{1}{24} \left[(2-3s) \frac{\partial}{\partial s} \right] \frac{\partial \mathcal{A}(s)}{\partial s} - \frac{7}{16\pi^2} \right. \\
&\quad \left. - (3w \frac{\partial}{\partial w} + 1) \frac{\partial \mathcal{A}(w)}{\partial w} \right] \mathcal{C}(v, f, t) + (1-2t) \frac{\partial}{\partial t} \frac{\mathcal{C}_1(v, f, t)}{384\pi^2} + \frac{1}{24} \left[s(t+4v+8w-4s) - tv - tw - 4w^2 \right] \\
&\quad \times \frac{\partial^2}{\partial t \partial w} + (s(8w-t-4s) - (4w-t)(w-v)) \frac{\partial^2}{\partial t \partial s} + 4t \frac{\partial}{\partial t} + 7 + (s-v-7w) \frac{\partial}{\partial w} + 3(s-w)(w \frac{\partial^2}{\partial w^2} \\
&\quad - s \frac{\partial^2}{\partial s^2}) + (3t-4s-v-2w) \frac{\partial}{\partial s} \mathcal{F}(v, f, t, s, w) \tag{A2} \\
\mathcal{W}_2(v, f, t, s, w) &= \frac{1}{72} \left[(1+2s) \frac{\partial}{\partial s} \frac{\partial \mathcal{A}(s)}{\partial s} + (1+2w) \frac{\partial}{\partial w} \frac{\partial \mathcal{A}(w)}{\partial w} \right] \mathcal{C}(v, f, t) + \frac{1}{48} \left[\left[\frac{2}{3} + (t-5s-w) \frac{\partial}{\partial s} \right] \right. \\
&\quad \times \frac{\partial \mathcal{A}(s)}{\partial s} + \left[\frac{2}{3} + (t-s-5w) \frac{\partial}{\partial w} \right] \frac{\partial \mathcal{A}(w)}{\partial w} - \frac{1}{3\pi^2} \left. \right] \frac{\partial \mathcal{C}(v, f, t)}{\partial t} + \frac{1}{72} \left[((4w-t-4s) \frac{\partial \mathcal{A}(s)}{\partial s} + (4s-t-4w) \right. \\
&\quad \times \frac{\partial \mathcal{A}(w)}{\partial w} - \frac{t}{4\pi^2}) \left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial v \partial t} \right) \mathcal{C}(v, f, t) + \frac{1}{576\pi^2} \left(\frac{\partial}{\partial t} - t \frac{\partial^2}{\partial t^2} - t \frac{\partial^2}{\partial v \partial t} + \frac{\partial}{\partial v} \right) \mathcal{C}_1(v, f, t) + \frac{1}{144} \left([2 + (w \right. \\
&\quad \left. - 7s - v) \frac{\partial}{\partial s}] \frac{\partial \mathcal{A}(s)}{\partial s} + [2 + (s-v-7w) \frac{\partial}{\partial w}] \frac{\partial \mathcal{A}(w)}{\partial w} - \frac{1}{2\pi^2} \right) \frac{\partial \mathcal{C}(v, f, t)}{\partial v} + \frac{1}{144} \left(5 \frac{\partial}{\partial t} \left(\frac{\partial}{\partial s} + \frac{\partial}{\partial w} \right) + 2(4w \right. \\
&\quad \left. - t) \frac{\partial^3}{\partial t^2 \partial s} + 3 \left(\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial w^2} + s \frac{\partial^3}{\partial s^3} + w \frac{\partial^3}{\partial w^3} \right) + 2(4s-t) \frac{\partial^3}{\partial t^2 \partial w} + 3(t-3s-w) \frac{\partial^3}{\partial t \partial s^2} + 3(t-s-3w) \right. \\
&\quad \times \frac{\partial^3}{\partial t \partial w^2} \left. \right) \mathcal{F}_1(f, w, s, t) + \frac{1}{144} \left(3 \frac{\partial^2}{\partial s \partial v} - \frac{\partial^2}{\partial s^2} + 3 \frac{\partial^2}{\partial w \partial w} - \frac{\partial^2}{\partial w^2} + (3s-v+w) \frac{\partial^3}{\partial s^2 \partial v} + (s-v+3w) \right. \\
&\quad \times \frac{\partial^3}{\partial v \partial w^2} \left. \right) \mathcal{F}_1(f, w, s, v) + \frac{1}{144} \left(4 \frac{\partial}{\partial t} - 4 \frac{\partial}{\partial s} - 4 \frac{\partial}{\partial w} - 3w(s-v+3w) \frac{\partial^3}{\partial w^3} + (3v-2s-t-7w) \frac{\partial^2}{\partial w^2} \right. \\
&\quad \left. + (3v-7s-t-2w) \frac{\partial^2}{\partial s^2} - 3s(3s-v+w) \frac{\partial^3}{\partial s^3} + 2[(8w-t-4s)s - (w-v)(4w-t)] \left(\frac{\partial^3}{\partial t \partial s \partial v} + \frac{\partial^3}{\partial t^2 \partial s} \right) \right. \\
&\quad \left. + (-5t+5v-28w) \frac{\partial^2}{\partial t \partial w} + 3[(4w-t-v+s)s - 5w^2 + tv + (t-3v)w] \frac{\partial^3}{\partial t \partial w^2} + (5v-28s-5t) \frac{\partial^2}{\partial t \partial s} \right)
\end{aligned}$$

$$\begin{aligned}
& +3((t-3v+4w-5s)s+(w-t)(w-v))\frac{\partial^3}{\partial t\partial s^2}+4\frac{\partial}{\partial v}+(3t-v-12w)\frac{\partial^2}{\partial v\partial w}+(3t-12s-v)\frac{\partial^2}{\partial v\partial s} \\
& +((t+v+8w-s)s-7w^2-tv+3tw-vw)\frac{\partial^3}{\partial v\partial w^2}+((3t-v+8w-7s)s-(w-t)(w-v))\frac{\partial^3}{\partial v\partial s^2} \\
& -4t\frac{\partial^2}{\partial t^2}-4t\frac{\partial^2}{\partial t\partial v}+2(-4s^2+(t+4v+8w)s-4w^2-tv-tw)(\frac{\partial^3}{\partial t^2\partial w}+\frac{\partial^3}{\partial t\partial w\partial v})\mathcal{F}(v,f,t,s,w) \quad (A3)
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_3(f,v,t,s,w,Q_{S_1},Q_{S_2}) &= (\frac{\partial\mathcal{A}(s)}{\partial s}+\frac{1}{8\pi^2})\frac{\mathcal{C}(f,v,t)}{24}+[(v-2s+2w)\frac{\partial\mathcal{A}(s)}{\partial s}+\frac{v}{8\pi^2}-14\mathcal{A}(s)+14\mathcal{A}(w)] \\
&\times\frac{\partial}{\partial v}\frac{\mathcal{C}(f,v,t)}{24}+\frac{1}{8}(\mathcal{A}(w)-\mathcal{A}(s))(2t\frac{\partial^2}{\partial v\partial t}+v\frac{\partial^2}{\partial v^2}+t\frac{\partial^2}{\partial t^2})\mathcal{C}(f,v,t)+\frac{1}{384\pi^2}[(w-s)(6t\frac{\partial^2}{\partial t\partial v}+3(v+t)\frac{\partial^2}{\partial t^2}) \\
&+1+(v-16s+16w)\frac{\partial}{\partial v}]\mathcal{C}_1(f,t,v)+\frac{1}{24}[(v-6s+2w)\frac{\partial}{\partial v}+1]\frac{\partial}{\partial s}\mathcal{F}_1(t,s,w,v)+\frac{1}{24}\left(1+(f+s-w)\frac{\partial}{\partial s}\right. \\
&+(v-16s+16w)\frac{\partial}{\partial v}+(f(v-6s+2w)-(2s-v-2w)(s-w))\frac{\partial^2}{\partial s\partial v}+3(w-s)[2t\frac{\partial^2}{\partial t\partial v}+v\frac{\partial^2}{\partial v^2}+t\frac{\partial^2}{\partial t^2}]) \\
&\times\mathcal{F}(f,v,t,s,w)+Q_{S_2}\left\{\frac{1}{24}[(1+3w\frac{\partial}{\partial w})\frac{\partial\mathcal{A}(w)}{\partial w}+\frac{1}{8\pi^2}]\mathcal{C}(f,v,t)+\frac{1}{6}[(\frac{1}{4}t-s+w)\frac{\partial\mathcal{A}(w)}{\partial w}+\frac{t}{32\pi^2}-\mathcal{A}(s)\right. \\
&+\mathcal{A}(w)]\frac{\partial}{\partial t}\mathcal{C}(f,v,t)+\frac{1}{384\pi^2}((-8s+t+8w)\frac{\partial}{\partial t}+1)\mathcal{C}_1(f,v,t)+[(\frac{1}{24}t-\frac{1}{6}s)\frac{\partial^2}{\partial t\partial w}-\frac{1}{8}w\frac{\partial^2}{\partial w^2}-\frac{5}{24}\frac{\partial}{\partial w}] \\
&\times\mathcal{F}_1(v,w,s,t)+\frac{1}{8}(w\frac{\partial}{\partial w}+2)\frac{\partial}{\partial w}\mathcal{F}_1(f,w,s,t)+\frac{1}{24}[1+(-5f-s+6v+7w)\frac{\partial}{\partial w}+(-8s+t+8w)\frac{\partial}{\partial t} \\
&+3w(-f-s+v+w)\frac{\partial^2}{\partial w^2}+(f(t-4s)+(4s-t-4w)(s-w))\frac{\partial^2}{\partial t\partial w}]\mathcal{F}(f,v,t,s,w)\Big\}+Q_{S_1}\{s\leftrightarrow w\} \quad (A4)
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_5(z,f,s,x,y) &= \mathcal{W}_{11}(z,f,s,x,y)+(-\frac{1}{24}\frac{\partial}{\partial x}+\frac{1}{3}\frac{\partial}{\partial y})\mathcal{F}_1(f,x,y,s)-\frac{1}{8}\frac{\partial}{\partial y}\mathcal{F}_1(f,y,x,z)+\left(\frac{1}{24}(-3s\right. \\
&+x-7y+8z)\frac{\partial}{\partial y}+\frac{1}{24}(-7x+y-z)\frac{\partial}{\partial x})\mathcal{F}(z,f,s,x,y) \\
\mathcal{W}_6(z,f,s,x,y) &= \mathcal{W}_{11}(z,f,s,y,x)+(\frac{5}{24}\frac{\partial}{\partial x}-\frac{5}{12}\frac{\partial}{\partial y})\mathcal{F}_1(f,x,y,s)+\frac{3}{8}\frac{\partial}{\partial y}\mathcal{F}_1(f,y,x,z)+\left(\frac{1}{24}(9s\right. \\
&+x-7y-10z)\frac{\partial}{\partial y}+\frac{1}{24}(-7x+y+5z)\frac{\partial}{\partial x})\mathcal{F}(z,f,s,x,y) \quad (A5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_7(z,f,s,x,y) &= \left(\frac{5}{36}\frac{\partial^2}{\partial x\partial s}-\frac{5}{72}\frac{\partial^2}{\partial y\partial s}+(\frac{y}{18}-\frac{s}{18})\frac{\partial^3}{\partial x\partial s^2}+(\frac{1}{36}s+\frac{1}{18}x)\frac{\partial^3}{\partial y\partial s^2}+\frac{1}{24}(s-x)\frac{\partial^3}{\partial y^2\partial s}\right. \\
&+\frac{1}{24}\frac{\partial^2}{\partial y^2}+\frac{1}{24}y\frac{\partial^3}{\partial y^3}-\frac{1}{8}x\frac{\partial^3}{\partial s\partial x^2})\mathcal{F}_1(f,x,y,s)+\frac{1}{72}\left((x+6y-z)\frac{\partial^3}{\partial y^2\partial z}+6\frac{\partial^2}{\partial y\partial z}-3\frac{\partial^2}{\partial x\partial z}-\frac{\partial^2}{\partial y^2}\right. \\
&-3x\frac{\partial^3}{\partial x^2\partial z})\mathcal{F}_1(f,x,y,z)+\frac{\mathcal{C}(z,f,s)}{12}(\frac{1}{3}+\frac{1}{2}y\frac{\partial}{\partial y})\frac{\partial^2\mathcal{A}(y)}{\partial y^2}+\frac{1}{144}\left((2-18x\frac{\partial}{\partial x})\frac{\partial\mathcal{A}(x)}{\partial x}+[2+(6s-6x\right. \\
&-12y)\frac{\partial}{\partial y}]\frac{\partial\mathcal{A}(y)}{\partial y}-\frac{1}{\pi^2})\frac{\partial\mathcal{C}(z,f,s)}{\partial s}+(\frac{\partial}{\partial z}-s\frac{\partial^2}{\partial s^2}+\frac{\partial}{\partial s}-s\frac{\partial^2}{\partial z\partial s})\frac{\mathcal{C}_1(z,f,s)}{576\pi^2}-\left(\frac{s}{288\pi^2}+\frac{1}{18}(s+x\right. \\
&-y)\frac{\partial\mathcal{A}(x)}{\partial x}-\frac{1}{36}(s+2x-2y)\frac{\partial\mathcal{A}(y)}{\partial y})\frac{\partial}{\partial s}(\frac{\partial}{\partial s}+\frac{\partial}{\partial z})\mathcal{C}(z,f,s)+\frac{1}{288}\left(-4(1+3x\frac{\partial}{\partial x})\frac{\partial\mathcal{A}(x)}{\partial x}+[12+(4x\right. \\
&-16y-4z)\frac{\partial}{\partial y}]\frac{\partial\mathcal{A}(y)}{\partial y}-\frac{1}{\pi^2})\frac{\partial\mathcal{C}(z,f,s)}{\partial z}+(\frac{1}{24}\frac{\partial}{\partial y}+\frac{1}{36}\frac{\partial}{\partial s}+\frac{1}{72}(-s-2x-7y+3z))\frac{\partial^2}{\partial y^2}-\frac{1}{24}y(x+y \\
&-z)\frac{\partial^3}{\partial y^3}-\frac{7}{72}\frac{\partial}{\partial x}+\frac{1}{72}(8s-3x-11y-5z)\frac{\partial^2}{\partial s\partial y}+\frac{1}{24}[x^2+(y-z)x-2y^2+s(-x+y+z)]\frac{\partial^3}{\partial s\partial y^2} \\
&+\frac{1}{72}(-13s-17x+3y+10z)\frac{\partial^2}{\partial s\partial x}+\frac{1}{36}[s(-x+y+z)-2(x^2-(2y+z)x+y^2)]\frac{\partial^3}{\partial s^2\partial y}+\frac{1}{18}(-x^2 \\
&-sx+2yx-y^2+sy-sz+yz)\frac{\partial^3}{\partial s^2\partial x}+\frac{1}{72}(6s-x-5y-3z)\frac{\partial^2}{\partial z\partial y}+\frac{1}{72}(s(x+6y-z)-(x-y)(x \\
&-4y-z))\frac{\partial^3}{\partial z\partial y^2}+\frac{1}{72}(-3s-7x+y+2z)\frac{\partial^2}{\partial z\partial x}+\frac{1}{36}[s(-x+y+z)-2(x^2-(2y+z)x+y^2)]\frac{\partial^3}{\partial z\partial y\partial s}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{36}s\frac{\partial^2}{\partial s\partial z} - \frac{1}{36}s\frac{\partial^2}{\partial s^2} + \frac{1}{36}\frac{\partial}{\partial z} + \frac{1}{18}(-x^2 - sx + 2yx - y^2 + sy - sz + yz)\frac{\partial^3}{\partial z\partial x\partial s} - \frac{1}{12}x^2\frac{\partial^3}{\partial x^3} \\
& -\frac{1}{8}x(x-y+z)\frac{\partial^3}{\partial x^2\partial s} - \frac{1}{24}x(s+x-y)\frac{\partial^3}{\partial z\partial x^2}\Big)\mathcal{F}(z, f, s, x, y) \\
\mathcal{W}_8(z, f, s, x, y) &= \frac{1}{8}\left[-\frac{\partial}{\partial x}\mathcal{F}_1(f, x, y, s) - \frac{\partial}{\partial y}\mathcal{F}_1(f, x, y, z) + \left(\frac{\partial\mathcal{A}(x)}{\partial x} - \frac{\partial\mathcal{A}(y)}{\partial y}\right)\mathcal{C}(z, f, s) + \left((x-s\right. \right. \\
& \left. -y)\frac{\partial}{\partial y} + (x-y-z)\frac{\partial}{\partial x}\right)\mathcal{F}(z, f, s, x, y)\Big] \\
\mathcal{W}_9(z, f, s, x, y) &= \frac{1}{8}\left[-\frac{\partial}{\partial x}\mathcal{F}_1(f, x, y, s) - \frac{\partial}{\partial y}\mathcal{F}_1(f, x, y, z) + \left(\frac{-\partial\mathcal{A}(x)}{\partial x} + \frac{\partial\mathcal{A}(y)}{\partial y}\right)\mathcal{C}(z, f, s) + \left((y-s\right. \right. \\
& \left. -x)\frac{\partial}{\partial y} + (-x+y-z)\frac{\partial}{\partial x}\right)\mathcal{F}(z, f, s, x, y)\Big]. \tag{A6}
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_{10}(z, f, s, x, y) &= \frac{1}{4}\left[\left(\frac{\partial\mathcal{A}(x)}{\partial x} - \frac{\partial\mathcal{A}(y)}{\partial y}\right)\left(\frac{\partial}{\partial s} + \frac{\partial}{\partial z}\right)\mathcal{C}(z, f, s) - \frac{\partial^2\mathcal{A}(x)}{\partial x^2}\mathcal{C}(z, f, s) - \left(\frac{\partial^2}{\partial x\partial s} + \frac{\partial^2}{\partial y\partial s} \right. \right. \\
& \left. + \frac{\partial^2}{\partial x^2}\right)\mathcal{F}_1(f, x, y, s) + \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} - 2y\frac{\partial^2}{\partial y^2} + (x+y-z)\frac{\partial^2}{\partial x^2} + (x-y-z)\left[\frac{\partial^2}{\partial y\partial s} + \frac{\partial^2}{\partial x\partial s} + \frac{\partial^2}{\partial y\partial z} \right. \right. \\
& \left. \left. + \frac{\partial^2}{\partial z\partial x}\right]\right)\mathcal{F}(z, f, s, x, y)\Big] \\
\mathcal{W}_{11}(z, f, s, x, y) &= \left(\frac{1}{6}(y-s)\frac{\partial^2}{\partial x\partial s} + \frac{1}{12}(s+2x)\frac{\partial^2}{\partial y\partial s} - \frac{1}{8}x\frac{\partial^2}{\partial x^2} + \frac{1}{8}y\frac{\partial^2}{\partial y^2}\right)\mathcal{F}_1(f, x, y, s) - (s\frac{\partial}{\partial s} + 1) \\
& \times \frac{\mathcal{C}_1(z, f, s)}{192\pi^2} - \frac{\mathcal{C}(z, f, s)}{24}\left((1+3x)\frac{\partial}{\partial x}\right)\frac{\partial\mathcal{A}(x)}{\partial x} + (1+3y)\frac{\partial}{\partial y}\frac{\partial\mathcal{A}(y)}{\partial y} + \frac{1}{4\pi^2}\Big) + \left(\frac{y-s-x}{6}\frac{\partial\mathcal{A}(x)}{\partial x} - \frac{s}{96\pi^2} \right. \\
& \left. + \frac{1}{12}(s+2x-2y)\frac{\partial\mathcal{A}(y)}{\partial y}\right)\frac{\partial\mathcal{C}(z, f, s)}{\partial s} + \left(\frac{1}{8}y(x-y+z)\frac{\partial^2}{\partial y^2} - \frac{1}{6}((s-2y+x)x + (y-s)(y-z))\frac{\partial^2}{\partial s\partial x} \right. \\
& \left. - \frac{1}{12} - \frac{1}{8}x(x-y+z)\frac{\partial^2}{\partial x^2} - \frac{1}{12}s\frac{\partial}{\partial s} + \frac{1}{12}((4y-s+2z-2x)x - 2y^2 + sy + sz)\frac{\partial^2}{\partial s\partial y}\right)\mathcal{F}(z, f, s, x, y) \tag{A7}
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_{12}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) &= \left(\frac{11}{32\pi^2} - \frac{\partial\mathcal{A}(x)}{\partial x}\right)\frac{\mathcal{C}(z, f, s)}{12} + \left[\frac{3s}{128\pi^2} - \frac{\mathcal{A}(x)}{2} + \frac{\mathcal{A}(y)}{2}\right]\frac{\partial}{\partial s}\mathcal{C}(z, f, s) + \frac{s}{8}\left[\frac{s}{32\pi^2} \right. \\
& \left. - \mathcal{A}(x) + \mathcal{A}(y)\right]\frac{\partial^2\mathcal{C}(z, f, s)}{\partial s^2} + \frac{1}{12}\left[(y-z-x)\frac{\partial\mathcal{A}(x)}{\partial x} + \frac{11z}{32\pi^2} - 7\mathcal{A}(x) + 7\mathcal{A}(y)\right]\frac{\partial\mathcal{C}(z, f, s)}{\partial z} + \frac{1}{8}\left[\frac{z}{32\pi^2} - \mathcal{A}(x) \right. \\
& \left. + \mathcal{A}(y)\right](2s\frac{\partial^2}{\partial z\partial s} + z\frac{\partial^2}{\partial z^2})\mathcal{C}(z, f, s) + \frac{1}{384\pi^2}\left(3(y-x)\left[z\frac{\partial^2}{\partial z^2} + 2s\frac{\partial^2}{\partial z\partial s} + 4\frac{\partial}{\partial s} + s\frac{\partial^2}{\partial s^2}\right] - 2 - 2(8x-8y+z) \right. \\
& \left. \times \frac{\partial}{\partial z}\right)\mathcal{C}_1(z, f, s) + \frac{1}{12}\left[(-3x+y-z)\frac{\partial^2}{\partial z\partial x} - \frac{\partial}{\partial x}\right]\mathcal{F}_1(f, x, y, z) + \left((-s-x+y)\left(\frac{1}{12}\frac{\partial}{\partial x} + \frac{1}{2}\frac{\partial}{\partial s} + \frac{1}{8}s\frac{\partial^2}{\partial s^2}\right) \right. \\
& \left. + \frac{1}{12}(8y-8x-7z)\frac{\partial}{\partial z} - \frac{1}{12}[(3s-2y+z-x)x + (y-s)(y-z)]\frac{\partial^2}{\partial z\partial x} - \frac{1}{8}(x-y+z)(2s\frac{\partial^2}{\partial z\partial s} + z\frac{\partial^2}{\partial z^2}) \right. \\
& \left. - \frac{1}{3}\right)\mathcal{F}(z, f, s, x, y) + Q_{F_2}\left[\left(-\frac{1}{3}\frac{\partial}{\partial y} + \frac{1}{12}(-s-2x)\frac{\partial^2}{\partial y\partial s} - \frac{1}{8}y\frac{\partial^2}{\partial y^2}\right)\mathcal{F}_1(f, y, x, s) + \frac{1}{4}\frac{\partial}{\partial y}\mathcal{F}_1(f, y, x, z) \right. \\
& \left. + \mathcal{C}(z, f, s)\left[\left(\frac{1}{8}y\frac{\partial}{\partial y} - \frac{1}{12}\right)\frac{\partial\mathcal{A}(y)}{\partial y} + \frac{1}{192\pi^2}\right] + \frac{1}{12}[(2y-s-2x)\frac{\partial\mathcal{A}(y)}{\partial y} + \frac{s}{16\pi^2} - 2\mathcal{A}(x) + 2\mathcal{A}(y)]\frac{\partial\mathcal{C}(z, f, s)}{\partial y} \right. \\
& \left. - \frac{\mathcal{C}_1(z, f, s)}{192\pi^2}\left[(s+4x-4y)\frac{\partial}{\partial s} + 1\right] + \left(\frac{1}{12}(3s+x+2y-4z)\frac{\partial}{\partial y} + \frac{1}{8}y(y-x-z)\frac{\partial^2}{\partial y^2} + \frac{1}{12}(4y-s-4x)\frac{\partial}{\partial s} \right. \right. \\
& \left. \left. + \frac{1}{12}[(s-4y-2z+2x)x + 2y^2 - sy - sz]\frac{\partial^2}{\partial s\partial y} - \frac{1}{12}\right)\mathcal{F}(z, f, s, x, y)\right] + Q_{F_1}\left[\left(\frac{1}{6} + \frac{1}{8}x\right)\frac{\partial\mathcal{A}(x)}{\partial x} + \frac{1}{192\pi^2}\right] \\
& \times \mathcal{C}(z, f, s) + \frac{1}{6}\left[(s+x-y)\frac{\partial\mathcal{A}(x)}{\partial x} + \frac{s}{32\pi^2} + \mathcal{A}(x) - \mathcal{A}(y)\right]\frac{\partial}{\partial s}\mathcal{C}(z, f, s) + \frac{\mathcal{C}_1(z, f, s)}{96\pi^2}\left[(s+2x-2y)\frac{\partial}{\partial s} + 1\right] \\
& + \left[\frac{1}{6}\frac{\partial}{\partial x} + \frac{1}{6}(s-y)\frac{\partial^2}{\partial x\partial s} + \frac{1}{8}x\frac{\partial^2}{\partial x^2}\right]\mathcal{F}_1(f, x, y, s) + \left(\frac{1}{12}(5x-2y+2z)\frac{\partial}{\partial x} + \frac{x}{8}(x-y+z)\frac{\partial^2}{\partial x^2} + \frac{1}{6}(s+2x \right. \\
& \left. - 2y)\frac{\partial}{\partial s} + \frac{1}{6}[x^2 + (s-2y)x + (y-s)(y-z)]\frac{\partial^2}{\partial s\partial x} + \frac{1}{6}\right)\mathcal{F}(z, f, s, x, y)\Big] \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_{13}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) &= W_{12}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) + Q_{F_1} \left(-\frac{1}{4} \frac{\partial}{\partial x} \mathcal{F}_1(f, x, y, s) - \frac{1}{4} \frac{\partial \mathcal{A}(x)}{\partial x} \mathcal{C}(z, f, s) \right. \\
&\quad \left. - \frac{\mathcal{C}_1(z, f, s)}{64\pi^2} + \frac{1}{4} [(y - x - z) \frac{\partial}{\partial x} - 1] \mathcal{F}(z, f, s, x, y) \right) + Q_{F_2} \left(-\frac{1}{4} \frac{\partial}{\partial y} \mathcal{F}_1(f, y, x, z) + \frac{1}{4} \frac{\partial \mathcal{A}(y)}{\partial y} \mathcal{C}(z, f, s) \right. \\
&\quad \left. + \frac{\mathcal{C}_1(z, f, s)}{64\pi^2} + \frac{1}{4} [(-s - x + y) \frac{\partial}{\partial y} + 1] \mathcal{F}(z, f, s, x, y) \right) \tag{A9}
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_{14}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) &= \left(\frac{\partial \mathcal{A}(x)}{\partial x} - \frac{7}{34\pi^2} \right) \frac{\mathcal{C}(z, f, s)}{6} + \left(\frac{\mathcal{A}(y)}{2} - \frac{3s}{128\pi^2} - \frac{\mathcal{A}(x)}{2} \right) \frac{\partial \mathcal{C}(z, f, s)}{\partial s} + \frac{s}{8} [\mathcal{A}(y) \\
&\quad - \mathcal{A}(x) - \frac{s}{32\pi^2}] \frac{\partial^2}{\partial s^2} \mathcal{C}(z, f, s) + \frac{1}{12} [(2z - x + y) \frac{\partial \mathcal{A}(x)}{\partial x} - \frac{7z}{32\pi^2} - 7\mathcal{A}(x) + 7\mathcal{A}(y)] \frac{\partial}{\partial z} \mathcal{C}(z, f, s) + \frac{1}{8} (\mathcal{A}(y) \\
&\quad - \frac{z}{32\pi^2} - \mathcal{A}(x)) [2s \frac{\partial^2}{\partial z \partial s} + z \frac{\partial^2}{\partial z^2}] + \frac{1}{384\pi^2} \left((y - x) [3z \frac{\partial^2}{\partial z^2} + 12 \frac{\partial}{\partial s} + 3s \frac{\partial^2}{\partial s^2} + 6s \frac{\partial^2}{\partial z \partial s}] + 4 + 4(-4x \right. \\
&\quad \left. + 4y + z) \frac{\partial}{\partial z} \right) \mathcal{C}_1(z, f, s) + [\frac{1}{12} (-3x + y + 2z) \frac{\partial^2}{\partial z \partial x} + \frac{1}{6} \frac{\partial}{\partial x}] \mathcal{F}_1(f, x, y, z) + \left(\frac{1}{6} (s + x - y) \frac{\partial}{\partial x} + (s - x \right. \\
&\quad \left. + y) [\frac{1}{2} \frac{\partial}{\partial s} + \frac{1}{8} s \frac{\partial^2}{\partial s^2}] + (-x + y + z) [\frac{2}{3} \frac{\partial}{\partial z} + \frac{1}{4} s \frac{\partial^2}{\partial z \partial s} + \frac{1}{8} z \frac{\partial^2}{\partial z^2}] + \frac{1}{12} [(2z - 3s + 2y - x)x - (y - s)(y \right. \\
&\quad \left. + 2z)] \frac{\partial^2}{\partial x \partial z} + \frac{5}{12} \right) \mathcal{F}(z, f, s, x, y) + Q_{F_2} \left([\frac{1}{6} + \frac{1}{8} y \frac{\partial}{\partial y}] \frac{\partial \mathcal{A}(y)}{\partial y} + \frac{1}{192\pi^2} \mathcal{C}(z, f, s) + \frac{1}{6} [(s - x + y) \frac{\partial \mathcal{A}(y)}{\partial y} \right. \\
&\quad \left. + \frac{s}{32\pi^2} - \mathcal{A}(x) + \mathcal{A}(y)] \frac{\partial}{\partial s} \mathcal{C}(z, f, s) + \frac{1}{96\pi^2} [(s - 2x + 2y) \frac{\partial}{\partial s} + 1] \mathcal{C}_1(z, f, s) + [\frac{1}{6} (s - x) \frac{\partial^2}{\partial y \partial s} + \frac{y}{8} \frac{\partial^2}{\partial y^2} \right. \\
&\quad \left. + \frac{5}{12} \frac{\partial}{\partial y}] \mathcal{F}_1(f, y, x, z) - \frac{1}{4} \frac{\partial}{\partial y} \mathcal{F}_1(f, y, x, z) + [\frac{1}{12} (5y - 3s - 2x + 5z) \frac{\partial}{\partial y} + \frac{1}{8} y (y - x + z) \frac{\partial^2}{\partial y^2} + \frac{1}{6} + \frac{1}{6} (s \right. \\
&\quad \left. - 2x + 2y) \frac{\partial}{\partial s} + \frac{1}{6} [y^2 - (s + 2y + z - x)x + sy + sz] \frac{\partial^2}{\partial y \partial s}] \mathcal{F}(z, f, s, x, y) \right) + Q_{F_1} \left([\frac{x}{8} \frac{\partial}{\partial x} - \frac{1}{12}] \frac{\partial \mathcal{A}(x)}{\partial x} \right. \\
&\quad \left. + \frac{1}{192\pi^2} \mathcal{C}(z, f, s) + \frac{1}{12} [(2x - s - 2y) \frac{\partial \mathcal{A}(x)}{\partial x} + \frac{s}{16\pi^2} + 2\mathcal{A}(x) - 2\mathcal{A}(y)] \frac{\partial}{\partial s} \mathcal{C}(z, f, s) - ((s - 4x + 4y) \frac{\partial}{\partial s} \right. \\
&\quad \left. + 1) \frac{\mathcal{C}_1(z, f, s)}{192\pi^2} + [\frac{1}{12} (-s - 2y) \frac{\partial^2}{\partial x \partial s} - \frac{1}{8} x \frac{\partial^2}{\partial x^2} - \frac{1}{12} \frac{\partial}{\partial x}] \mathcal{F}_1(f, x, y, s) + [\frac{1}{12} (2x + y - z) \frac{\partial}{\partial x} + \frac{1}{8} x (x - y \right. \\
&\quad \left. - z) \frac{\partial^2}{\partial x^2} + \frac{1}{12} (-s + 4x - 4y) \frac{\partial}{\partial s} + \frac{1}{12} [2x^2 - (s + 4y)x + (s + 2y)(y - z)] \frac{\partial^2}{\partial x \partial s} - \frac{1}{12}] \mathcal{F}(z, f, s, x, y) \right) \tag{A10}
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}_{15}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) &= W_{14}(z, f, s, x, y, Q_{F_1}, Q_{F_2}) + Q_{F_1} \left(\frac{1}{4} \frac{\partial \mathcal{A}(x)}{\partial x} \mathcal{C}(z, f, s) - \frac{1}{4} \frac{\partial}{\partial x} \mathcal{F}_1(f, x, y, s) \right. \\
&\quad \left. + \frac{\mathcal{C}_1(z, f, s)}{64\pi^2} + \frac{1}{4} [(x - y - z) \frac{\partial}{\partial x} + 1] \mathcal{F}(z, f, s, x, y) \right) + Q_{F_2} \left(-\frac{1}{4} \frac{\partial}{\partial y} \mathcal{F}_1(f, y, x, z) - \frac{1}{4} \frac{\partial \mathcal{A}(y)}{\partial y} \mathcal{C}(z, f, s) \right. \\
&\quad \left. - \frac{\mathcal{C}_1(z, f, s)}{64\pi^2} + \frac{1}{4} [(-s + x - y) \frac{\partial}{\partial y} - 1] \mathcal{F}(z, f, s, x, y) \right) \tag{A11}
\end{aligned}$$